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Global sea level trends in the presence of variable sea level velocities, and variable accelerations

Research Article

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Abstract:
This study investigates, using a new *variable-acceleration* model, the validity of the implicit assertion in previous studies regarding global constant sea level rise accelerations. Thirteen out of twenty seven globally distributed tide gauge stations, with records longer than 80 years, exhibit statistically significant quartic coefficients ($p < 0.05$) revealing the presence of variable sea level accelerations though not as a global phenomenon. Most of these stations initially exhibit decreasing negative velocities until early 20th century and increasing positive velocities after 1970's following a period of constant velocities. It is shown that, for those locations experiencing statistically significant variable sea level accelerations, the estimates based on the conventional linear representation of linear sea level trends are not appropriate, and are notably biased for a number of stations. All solutions account for serial correlations, which otherwise induce biases in solution statistics. It is also demonstrated that the omission of non-linearities in sea level changes will bias the sea level trends for short records, such as those from satellite altimetry, as large a 3 mm/yr.

Keywords:
Climate change • sea level rise • satellite altimetry • tide gauge • variable acceleration • variable velocity

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A trend is a trend is a trend
But the question is, will it bend?
Will it alter its course
Through some unforeseen force
And come to a premature end?

Caincross (1969)

1. Introduction

One of the serious consequences of global warming is an increase in sea levels. Although sea levels exhibit natural spatial and temporal variability ranging from hourly to millennial scales, accurate determination of sea level trends and a possible acceleration during the last century is needed to discern anthropogenic contributions to sea level rises.

Among a multitude of early investigations as early as 1990, Woodworth reported "an overal slightly negative acceleration observed in northern Europe". His analysis with extended series to timescales longer than a century, showed a positive acceleration 0.4 mm/yr/century for the European Atlantic coast and Baltic sea level. A study by Douglas (1992) reported also -0.011 mm/yr/yr negative acceleration estimated from globally distributed tide gauge data. Over a decade later, Holgate and Woodworth (2004) reported a sea level rise over the last 55 years of about 1.7 ± 0.2 mm/yr, based on 177 tide gauges divided into 13 regions. Their altimetry data analysis showed a rate of sea level rise around the global coastline significantly in excess of the global average over the period 1993–2002. They also reported that the globally-averaged rate of coastal sea level rise for the same decade centered on 1955 was significantly

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larger than any other decade during the past 55 years. A study by Church and White (2006) revealed 0.013 mm/yr/yr positive acceleration in sea level rise. A more recent research by Holgate (2007), from the analysis of nine long tide gauge records during 1904–2003, reported that the rate of sea level change was larger (2.03 $\pm$ 0.35 mm/yr) in the early part of last century (1904–1953), in comparison with the latter part (1954–2003) 1.45 $\pm$ 0.34 mm/yr. The results from the analysis of a 300 year long global sea level using two different methods by Jevrejeva et al. 2008 suggested global sea level acceleration up to the present has been about 0.01 mm/yr/yr. On the other hand, in 2009, the International Panel on Climate Change, stated much faster sea level rise of 3.1 mm/year during 1993-2003 as compared with 1.8 mm/yr during 1961-2003 (IPCC, WG1, Table SPM.1). Later, Woodworth et al. (2009) reported that most sea-level data originate from Europe and North America, and that both sets display evidence for a positive acceleration around 1920-1930 and a negative acceleration around 1960. Whereas two other recent studies by Houston and Dean (2011), and Watson (2011), using tide gauge data showed no acceleration globally and in fact, a negative acceleration for the regional (Australia) sea level rise.

The above list of estimates about the sea level trends at secular time scales and a possible acceleration in sea level rise is by no means exhaustive. However, the plethora of divergent estimates about the sea level rise inferred from tide gauge and satellite altimetry data suggest that sea level accelerations may also be variable as a function of time and location, a possibility that was recognized as decadal and interdecadal variations but was not vigorously formulated and scrutinized in earlier investigations. *Therefore, the aim of this study is to investigate the existence of time varying sea level accelerations in globally distributed long tide gauge records as a global phenomenon using a new variable sea level acceleration model and their impact on average sea level velocity estimates.* *This study is also the first study that accounts for the serial correlations in tide gauge data in estimating solution statistics in globally distributed long tide gauge records, which, if not modeled, bias solution statistics.* In the following section, first, a time varying sea level acceleration model is presented using a new quartic representation of the sea level changes with a kinematic interpretation.

In the presence of variable sea level accelerations, the sea level trends are dependent on the initial epoch of the records (initial velocities). *Hence, an new definition of the secular trend is introduced for the records that exhibit variable accelerations in estimating sea level rise studies.*

The subsequent section describes an extension of the quartic model that also includes periodic seal level variations within the framework of a statistical model that accounts for serially correlated tide gauge data.

In what follows is the description of tide gauge records used in the study and their analysis based on the new model, referred here as *variable-acceleration model*. In addition, trend estimates were also calculated using linear trend only models with no accelerations, referred here as *no-acceleration models*, which were used as baselines in evaluating the impact of the variable sea level accelerations on the estimates for the average sea level velocities.

2. Variable Sea Level Velocity and Acceleration Model

Let the sea level acceleration $\ddot{h}_t$ at an epoch t be represented by the following time varying quadratic model:

$$\ddot{h}_t := \frac{d^2 h_t}{dt^2} = b + ct + dt^2 \tag{1}$$

where h_t is the sea level height at an epoch t, and b, c, and d are the coefficients of the quadratic. The polynomial representation is restricted to a quadratic due to the fact that higher frequency variations in sea level accelerations are not likely to have a marked impact on long time series and also due to the fact that the potential harmful collinearity the finalized model may experience as a result of any higher degree terms. The integration of the quadratic (1) gives the velocity model:

$$\dot{h}_t := \frac{dh_t}{dt} = \dot{h}_{t_0} + bt + c\frac{t^2}{2} + d\frac{t^3}{3} \tag{2}$$

and the integration of the above velocity model gives the following *quartic* model to represent the nonlinear and aperiodic sea level changes;

$$h_t = h_{t_0} + \dot{h}_{t_0} t + b\frac{t^2}{2} + c\frac{t^3}{6} + d\frac{t^4}{12} \tag{3}$$

In these expressions, $\dot{h}_{t_0}$, denotes the velocity, which is the initial rate of change in the sea level rise at a predefined epoch t_0. The velocity $\dot{h}_t$ varies in time in the presence of sea level accelerations and, h_{t_0} is the height of the sea level at the initial epoch t_0.

It is important to note that in the presence of *variable-accelerations*, the velocity (2) is no longer *invariant* throughout the series. Hence, the following time average of the change in sea level height over the series span is proposed to represent the rate of secular rise in sea level,

$$\bar{\dot{h}} := \frac{h_{t_{End}} - h_{t_{Start}}}{t_{End} - t_{Start}} \tag{4}$$

in which $\bar{\dot{h}}$ is the average trend/velocity by definition $h_{t_{Start}}$ and $h_{t_{End}}$ refer to the sea level heights at the beginning and at the end of the series denoted by t_{Start} and t_{End} respectively. Again $\bar{\dot{h}} = \dot{h}_{t_0}$ if, $b = c = d = 0$.

Early velocity-only (trend models), and recent fixed-acceleration models are all empirical and descriptive in nature hence yet to be informative in explaining the underlying phenomena. The proposed *variable-acceleration* model, though it extends the representation of sea level variations to accommodate also the aperiodic changes, has similar limitations. Nonetheless, *variable-acceleration* models are useful for detailed exploration of the kinematics of the sea level changes. Third order derivatives of the quartic models gives *jerks*, and their forth order derivatives give *spasms*, concepts yet to be recognized by the sea level community.

3. Variable-acceleration Models with Periodic Sea Level Changes

Sea level heights are also subject to a series of short periodic variations. The use of monthly averaged tide gauge data significantly reduces the variability of the records. Yet, the annual and semi-annual periodic sea level changes carry high power and should be represented in the model to improve its explanatory power thereby, for accurate testing of the significance of the quartic model coefficients and the predictive power of the model itself.
In addition, periodic lunar node tides also need to be represented in the model. Albeit their negligible contributions to the solutions from long time series, lunar node tides correlate well with the coefficients of the following proposed quartic model. Their omission, hence, will bias the estimates of the quartic model parameters. The mixed kinematic model of sea level variations is therefore given by:

$$\begin{aligned} h_t =& h_{t_0} + \dot{h}_{t_0}(t-t_0) + b\frac{(t-t_0)^2}{2} + c\frac{(t-t_0)^3}{6} + d\frac{(t-t_0)^4}{12} \\ &+ \sum_{h=1}^{3}\left[\alpha_h \cos\left(\frac{2\pi}{P_h}(t-t_0)\right) + \gamma_h \sin\left(\frac{2\pi}{P_h}(t-t_0)\right)\right] \end{aligned} \quad (5)$$

In this expression h_t represents the monthly averaged tide gauge data available in between starting and ending epochs, $t = t_{Start} \cdots t_{End}$, and, h_{t_0} is the unknown sea level reference height defined at the initial epoch of the measurements t_0, which is now defined at the middle of the series. The coefficients b, c, and d are the unknown quartic coefficients that can be determined using a least square solution from the tide gauge records. The additional unknown coefficients of the sine and cosine terms are denoted by α and γ from which the amplitudes and the phase angles of semi-annual, annual, and lunar node (18.613 yr.) periods, P_h, are determined.
The time component of the quartic's d coefficient grows with the fourth power of the epoch of measurement and may cause harmful collinearity in least squares solutions especially for long series. Shifting the initial epoch of the measurements to the middle of the series switches the epoch of the observations to time differences, which are smaller in magnitude, hence improves the stability of the solutions. In this case, the initial velocity $\dot{h}_{t_0}$ will refer to the middle of the series.
This model also accommodates earlier and recent sea level rise models if the solution dictates; i.e. if the estimate for the d coefficient in Eq. (1) is not statistically significant, then the quartic representation reduces to a *linear-acceleration* model. If the c coefficient is zero, as a result of null-hypothesis testing, the resulting model is a *constant-acceleration* model.
Meanwhile, all kinematic models with high order polynomial representations should be carefully used for forecasting sea level trends, including the proposed quartic representation. A quartic model, being an even degree polynomial, can also conveniently represent rapid changes in sea level heights at the beginning and at the end of the series as revealed by the sign of the d coefficient, but at the same time the error estimate for the extrapolated values will increase in the vicinity and beyond the series with the increasing degree of the polynomial extrapolation (Runge's phenomenon, Runge, 1901).
Moreover, a quartic (bi-quadratic) model exhibits sixteen distinct combinations of sign ordering of its coefficients, sufficient enough for capturing long term periodic, episodic, deterministic, or stochastic excursions (at decadal and interdecadal scales) regardless of their origins in sea level heights, thereby reducing the effect of unmodeled variations on the secular trends, and instrumental for searching a global sea level acceleration.

4. Stochastic Model

Traditionally, the stochastic model involved in sea level studies assumes that the *random* variable e_t (disturbances), which represents the lump-sum effect of the instrument errors and the lump-sum unmodeled effects, are uniform (homogenous) and independent of each other (uncorrelated). A recent study by Iz et al. (2012), revealed that the disturbances exhibit autoregressive behavior of the first order for the tide gauge stations used in that study with a *positive* serial correlation coefficient as large as 0.4. *Durbin-Watson* tests using the residuals of the preliminary solutions for the 27 stations involved in this study also confirmed the presence of statistically significant first order autoregressive processes in *all* tide gauge stations.
If the disturbances $e_t$at a given station are sequentially interdependent ignoring serial correlations does not bias the estimated parameters. Yet their omission can cause overestimation of the accuracy of the estimated parameters (Neter et al., 1996) if the correlation coefficients are positive, thereby introduce spurious parameters as significant, which are otherwise rejected in null-hypothesis testing for their significance.
It is rather surprising that, despite the recognition of the serial correlation in tide gauge records (Maul and Martin, 1993), as part of decadal and interdecadal sea level variations, not very many studies have accounted for them in estimating model statistics, such as standard errors of the estimates and the R^2 values in global sea level studies.
The following first order autoregressive process can represent the stochastic behavior of the disturbances:

$$e_t = \rho e_{t-1} + v_t \quad (6)$$

where, ρ represents the correlation between e_t and e_{t-1} at two subsequent epochs $t-1$ and t. The stochastic process v_t at the epoch t has the following *assumed* properties,

$$E(v_t) = 0, \qquad E({v_t}^2) =: {\sigma_t}^2, \qquad E(v_t v_{t'}) = 0, \qquad t \neq t' \tag{7}$$

which yields the variance of the disturbances, $\sigma^2_{e_t}$, as follows,

$$E[e_t] = 0, \quad Var[e_t] = {\sigma_v}^2(1-\rho^2)^{-1} = {\sigma_{e_t}}^2. \tag{8}$$

If the autocorrelation coefficient is positive, which is the case for all the stations analyzed in this study, then the Hildreth - Lu procedure (Hildreth and Lu, 1960), which is based on a simple transformation, can be applied to estimate the model parameters. First differencing of successive tide gauge data eliminates the autocorrelated portion of the disturbances in Eq. (6). Differencing affects only the intercept parameter (which is recovered after the solution) leaving the other regression parameters invariant. The model parameters can then be estimated using a number of Ordinary Least Squares solutions using regularly sampled correlation coefficients that appear in the transformed data within the interval [-1, 1]. The solution that gives the largest R^2 value is then selected as the best fit autocorrelation coefficient and its solution as the terminal solution. An alternative approach was also used by Iz and Chen 1999, and Iz et al. 2012.

Once the model parameters are estimated using the above statistical model, the secular sea level trend can be obtained from the newly defined average velocity given by Eq. (4) using the model based predicted heights at the beginning $\hat{h}_{t_{start}}$ and at the end of the series $\hat{h}_{t_{end}}$;

$$\hat{\bar{\dot{h}}} := \frac{\hat{h}_{t_{end}} - \hat{h}_{t_{start}}}{t_{end} - t_{start}}. \tag{9}$$

In determining average velocities, it is preferable to calculate the predicted heights using only the quartic model parameters leaving out the periodic effects for robust estimates. While the omission of periodic variations on the average velocity is negligible for long series, the average velocity for shorter series will be different as a function of the length of the series if the periodic changes are included in the above expression.

All the average velocity estimates reported in this study follows this guideline regardless of the record length, i.e. all model parameters, inclusive of the coefficients of the quartic and periodic components were first estimated. The adjusted starting and ending sea level heights at the starting and ending epochs were then calculated using quartic parameters only, leaving out the periodic components, to estimate the average velocity given by Eq. (9).

The standard error of the error average secular rate is calculated using the error estimates of the predicted heights at the beginning and end of the series by variance propagation.

5. Globally Distributed Long Tide Gauge Time Series

Permanent Mean Sea Level (PSMSL) repository maintains a tide gauge database from over 1800 stations since 1933. PSMSL offers Metric and Revised Local Reference (RLR) data (PSMSL, 2011). The metric data is the raw data directly received from the authorities, whereas the RLR data contains monthly and annual MSL data referenced to a common datum. Given the fact that the longer the series are, the more robust the estimates of the sea level trend are against unmodeled sea level variations (Iz, 2006), only stations that span close to and over a century were deployed in this investigation. The RLR tide gauge data, downloaded in April 2011 and listed in Table 1, are used in this study.

Tide gauge data (or estimated trend estimates) were not corrected for the effect of the post glacial rebound, hence all inferences refer to *relative sea level changes*.

6. Solutions

Solutions using two different models were considered. The *no-acceleration* model is the typical trigonometric model in estimating linear sea level trends in current literature. It is also a subset of the proposed *variable-acceleration* model in which the polynomial terms $b = c = d = 0$.

The *no-acceleration* model is to evaluate the impact of the *variable-acceleration* models on various estimated parameters and their statistics. However, as opposed to the current practice, the observation errors, in this study, are recognized to follow the same first order autoregressive process, which was discussed in the previous section for the *no-acceleration* solutions.

Hildreth-Lu (1960) procedure was used to estimate the trend parameters as well as semi-annual, annual, and node parameters for the *no-acceleration* model. Estimated parameters were subjected to t-tests for their significance, and those parameters with $p > 0.05$ were removed from the models and the new solutions were obtained for the reduced models. Concurrently, each model solution was subjected to F-tests for their predictive powers. All solutions passed their F-tests at 0.05 significance level, including those with low R^2 values thanks to their degrees of freedom that can be as large as 1200.

Some of the pertinent statistics are listed in Table 1. The first line for each station consists of the length of the series, the estimated linear trend, *velocity*, and its variance, the *root mean square error, RMSE* of the solution, and the *adjusted coefficient of determination*, $R^2(adj)$.

The second line for each station in Table 1 shows all the estimates for the *variable-acceleration* model Eq. (3) and their uncertainties including the initial velocity and the average velocity calculated using Eq. (9). Hildredth-Lu (1960) iterative procedure was applied for the least squares solutions with autocorrelated disturbances. The autocorrelation coefficient that gives the largest R^2 determined by the procedure for each station were within the range of 0.20 – 0.40, overwhelmingly concentrated about 0.30.

Table 1. The solution statistics for *no-acceleration* (the first lines) and *variable-acceleration* models. Records are in years, velocities in mm/yr followed by their standard errors. The *RMSE* is in mm, and adjusted R^2 values are in percent. Stations with statistically significant *variable-acceleration parameters* ($p < 0.05$) are shown in red. NA: Not Applicable. No PGR corrections were applied. Estimates for the reference heights, amplitudes of semi-annual, annual and node periods are not listed.

Station	*Span*	$\hat{h}_{t_0}$	σ	$\hat{\dot{h}}$	σ	$\hat{b}$	σ	$\hat{c}$	σ	$\hat{d}$	σ	*RMSE*	$R^2(adj)$
AU Sydney*	96			0.91	0.10	NA		NA		NA		50	18.4
		1.51	0.18	0.52	0.58	0.063	0.021	-0.0026	0.007	-0.00020	0.00006	47	26.4
AU Sydney	108			0.57	0.05	NA		NA		NA		51	31.7
		0.57	0.05	0.56	0.05	0.096	0.015			-0.00018	0.00003	47	26.0
CA Ketchikan	91			-0.18	0.12	NA		NA		NA		74	39.4
				-0.18	0.12							74	39.4
CA Prince Rupert	101			1.07	0.14	NA		NA		NA		73	38.4
		1.72	0.25	0.82	0.51			-0.0021	0.0007			72	43.7
DE Cuxhaven	165			2.53	0.07	NA		NA		NA		146	47.3
				2.54	0.10							154	23.9
DE Den Helder	146			1.48	0.10	NA		NA		NA		106	11.3
				1.48	0.10							106	11.3
DE Travemunde	154			1.66	0.06	NA		NA		NA		80	34.8
				1.66	0.06							80	34.8
FR Brest	202			1.05	0.05	NA		NA		NA		78	19.8
				1.05	0.05							78	19.8
IN Mumbai	129			0.76	0.06	NA		NA		NA		52	26.2
		1.52	0.12	-0.07	0.50	-0.075	0.009	-0.0023	0.0003	0.00012	0.00002	51	33.0
NL Amsljmuiden	140			1.61	0.08	NA		NA		NA		92	32.7
		2.60	0.17	0.95	0.18			-0.0020	0.0003	0.00004	0.00001	90	41.6
NL Delfzijl	146			1.67	0.07	NA		NA		NA		117	33.8
		1.67	0.07	1.67	0.07					0.00002	0.00000	116	34.3
NL Harlingen	146			1.38	0.10	NA		NA		NA		127	9.7
				1.38	0.10							127	9.7
NL Terschelling	90			1.02	0.13	NA		NA		NA		102	31.3
		0.06	0.33	0.06	0.46			0.0048	0.0015			102	31.9
PL Swinoujscie	189			0.81	0.06	NA		NA		NA		103	15.9
				0.81	0.06							103	15.9
SE Landsort	122			-2.85	0.15	NA		NA		NA		125	29.2
				-2.85	0.15							125	29.2
SE Stockholm	121			-3.83	0.16	NA		NA		NA		127	36.0
				-3.83	0.16							127	36.0
UK Liverpool	126			1.06	0.10	NA		NA		NA		88	25.9
				1.06	0.10							88	25.9
UK N. Shields	115			1.89	0.07	NA		NA		NA		56	53.3
		1.85	0.07	1.85	0.07	-0.044	0.015			0.00009	0.00003	56	53.5
US Annapolis	81			3.42	0.13	NA		NA		NA		54	65.0
		2.73	0.26	2.73	0.39			0.0042	0.0014			54	70.4
US Atlantic City	98			4.04	0.09	NA		NA		NA		58	70.2
				4.04	0.09							58	70.2
US Baltimore	107			3.08	0.07	NA		NA		NA		55	74.8
		3.08	0.07	3.08	0.07	-0.076	0.018			0.00018	0.00004	54	75.1
US Boston	89			2.65	0.09	NA		NA		NA		45	48.7
		1.58	0.19	3.36	0.27	-0.102	0.023	0.0054	0.0009	0.00025	0.00008	44	58.3
US Honolulu	105			1.47	0.10	NA		NA		NA		33	28.3
				1.47	0.10							33	28.3
US Fernandina	112			2.03	0.10	NA		NA		NA		76	57.3
		2.20	0.11	2.20	0.11					0.00004	0.00001	76	57.6
US Key West	97			2.14	0.18	NA		NA		NA		42	67.3
				2.14	0.18							42	67.3
US New York	154			2.80	0.05	NA		NA		NA		56	74.5
				2.80	0.05							56	74.5
US Pensacola	87			2.14	0.13	NA		NA		NA		50	54.4
		1.31	0.31	2.70	0.44			0.0044	0.0015			50	54.7
US San Francisco	155			1.42	0.08	NA		NA		NA		46	27.3
		1.42	0.06	1.42	0.06	0.060	0.010			-0.00006	0.00001	45	35.5

*Fort Denison, station ID: 196

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Again, as before, those *variable-acceleration* model parameters whose t-scores with $p > 0.05$ were removed and new solutions were generated using the reduced models. The same full F-tests were carried out to check the predictive power of the models, which were all significant at 0.05 level.

Variance inflation factors, VIF, were calculated for all the estimates (regression of each one of the parameters, as dependent variable, and the others as independent variables) to check multiple collinearity especially for correlations among quartic parameters. For those parameters that pass the t tests, all the $VIFs$ were less than two, indicative of negligible correlations among the parameters.

$R^2(adj)$ values were larger and the $RMSE$ were smaller for the *variable-acceleration* model solutions with serially correlated (autocorrelated) disturbances than those of the baseline model solutions (*no-acceleration* models), as expected. In some cases the improvements were not impressive and in some others, there were no improvements because, the *no-acceleration* and the *variable acceleration* models overlap as a result of rejecting all the quartic model coefficients under the null-hypothesis testing.

7. Model Verification

Two neighboring tide gauge stations in Sydney Australia, Fort Denison and Denison 2, demonstrate the reproducibility of *variable-acceleration* model solutions as compared to *no-acceleration* model solutions (the first two entries in Table 1). These two stations are within a few km from each other with data span of 108 and 96 years respectively. Because of their proximity, they sample the same environment. Notwithstanding the variability in their data, trend estimates from these two stations are expected to be the same.

No-acceleration model trend estimates for the two nearby Fort Denison stations in Sydney in Table 1 are markedly different, 0.91 ± 0.10 vs. 0.57 ± 0.05 mm/yr. On the other hand, the average sea level trend estimates based on *variable-acceleration* model, are 0.52 ± 0.58 vs. 0.56 ± 0.05, in agreement despite the large uncertainty of the Fort Denison 2 station estimate, which can be due to poor separability of long variations from the trend for this station. The magnitudes of the estimated quartic coefficients for both stations are also in agreement in magnitude and direction.

The adjusted R^2 for both stations are also consistent (26.4 and 26.0 percent) for the *variable-acceleration model* solutions despite the differences in series lengths. The $RMSE$ values for the *variable-acceleration* models not only consistent but also smaller than the *no-acceleration* solution's values (47 and 47 mm versus 50 and 51 mm).

Observe that the significant differences in the initial velocities for the variable-acceleration solutions support the necessity for the proposed average velocity formulation.

8. Analysis of Results

Despite the improved $R^2(adj)$ values, only thirteen out of twenty seven globally distributed tide gauge stations exhibit statistically significant ($p < 0.05$) quartic coefficients revealing the presence of variable sea level accelerations in these records. Among the thirteen stations, eight stations' solutions show marked difference in magnitude between average rates obtained from *variable-acceleration* and velocities from *no-acceleration* models.

Solutions with statistically significant *variable-accelerations*, the adjusted sea level heights, *variable velocities* and *variable accelerations* referenced to the middle of the series are depicted in Figure 1 (estimated intercepts are set to zero for clarity).

All the tide gauge stations used in this study with significant quartic parameters, with the exception of San Francisco, (USA) and Prince Rupert (Canada), exhibit increased positive velocities toward the end of the series after 1970s, which are starkly replicated on the *variable-acceleration* plots (third plot in Figure 1 for each station). The *variable-acceleration* plots reveal that majority of the tide gauges stations analyzed in this study experience a decreasing negative acceleration during the beginning of the series, followed by an oscillating velocity period until 1960s, after which there is a clear increasing positive acceleration in sea level rise. It should be emphasized that this behavior cannot and should not be generalized for a global synthesis because of limited sampling.

9. The Effect of Variable Acceleration on Secular Trends from Shorter Records

The magnitudes of most of the averaged velocities calculated using the *variable-acceleration* models are different than those estimated from *no-acceleration* models. Their presence also implicates that they will bias the velocity estimates from shorter records, such as satellite altimetry in the same region, if they are not accounted for in analyzing these records.

To illustrate the effect of biasing, consider modeling satellite records overlapping with the last 20 years of the Sydney (AU) series, during 1915 – 2010, for estimating the sea level rise during these two decades towards the end of the series. If the following model, which ignores the presence of the variable accelerations given by Eq. (5), is used for estimating the sea level trends (the periodic model parameters can be safely ignored for the sake of simplicity),

$$h_t = h_{t_0} + \dot{h}_t(t - t_0) + e_t \tag{10}$$

then, Δ_t, the omission error, is given by;

$$\Delta_t = b\frac{(t-t_0)^2}{2} + c\frac{(t-t_0)^3}{6} + d\frac{(t-t_0)^4}{12}. \tag{11}$$

Following the derivation steps given in Iz (2006) for the unmodeled effects, it can be shown that the trend estimate, $\hat{\dot{h}}_t$ from the satellite altimetry solution using Eq. (10) will be biased if $E(\Delta_t) \neq 0$.

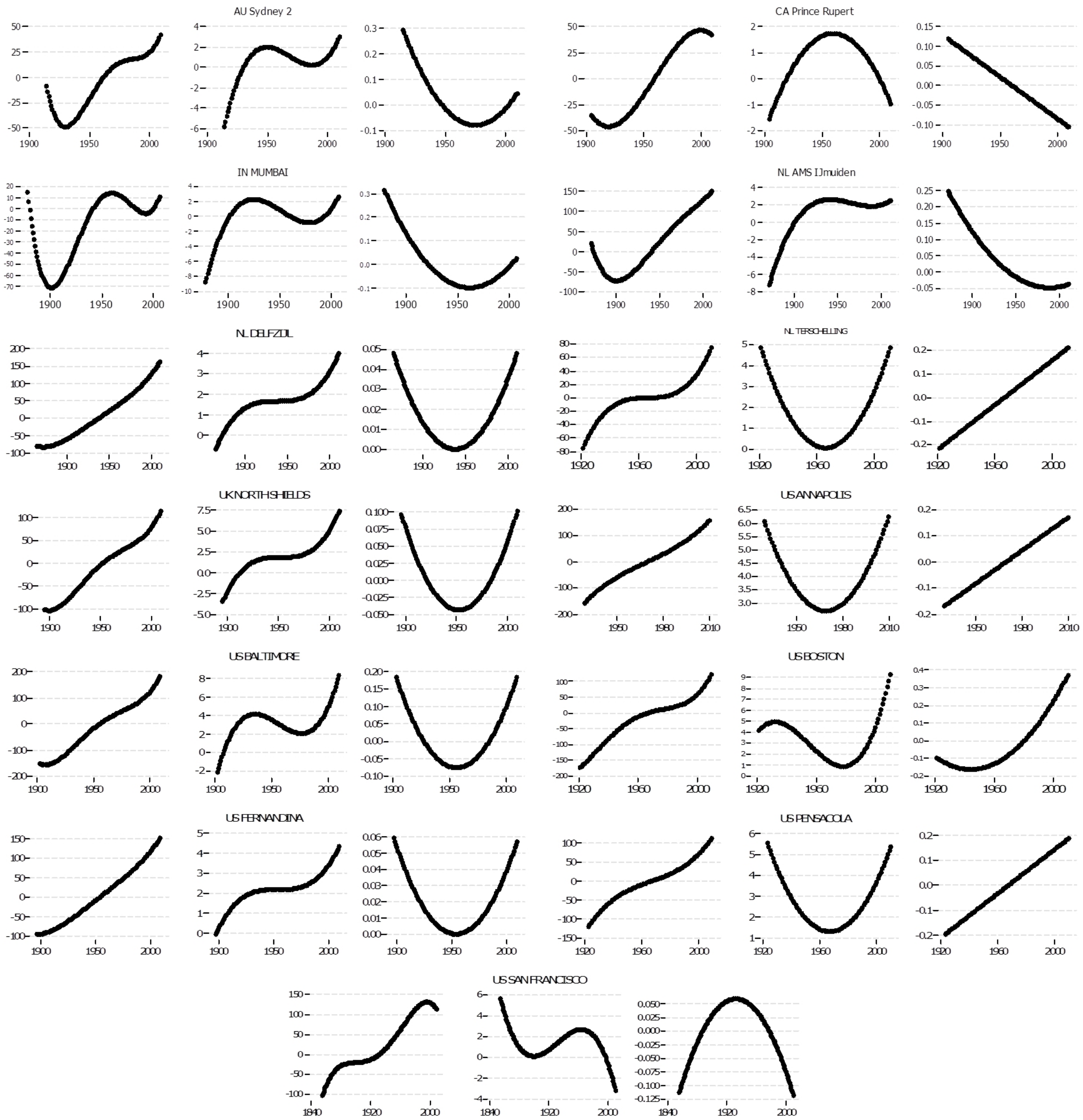

Figure 1. In the first graph of each station, the vertical axes show estimated sea level heights using quartic models in mm referenced to zero intercept. The second graph of each station display the display the sea level velocities in mm/yr which is generated from the estimated quartic parameters using the velocity equation. The accelerations (mm/yr/yr) of each stations are based on the acceleration equation with estimated quartic parameeters. All plots are generated during each stations' record periods at yearly intervals. Horizontal axes are in years and the initial period of each series are defined at the middle epoch of the series listed in Table 1.

The bias can be computed using the following expression,

$$bias := \dot{h}_t - E(\hat{\dot{h}}_t) = \frac{\sum_t \Delta_t (t - \bar{t})}{\sum_t (t - \bar{t})^2} \qquad (12)$$

where circumflex denotes the estimated velocity.

If the time variable t runs from 1990 through 2010 at monthly intervals, and, $\bar{t}$ is the reference epoch 2000 for the satellite altimetry measurements, and $t_0 = 1962$ is the reference epoch for the tide gauge data, then the calculated bias, using Eq. (12), is -3.36 mm/yr, a marked correction for the estimated velocity from satellite altimetry records in this region.

The first panel of Figure 1 for this station shows that the last 20 years of tide gauge records were subject to a considerable upward trend as part of a variable acceleration of local effect, therefore, the correction is consistent, and will reduce the satellite altimetry rate closer to the expected level of 0.52 mm/yr listed in Table 1 inferred from the long records using the average velocity model.

9.1. Conclusion

The new quartic kinematic and statistical models with first order autoregressive disturbances (serial correlation) introduced in this study detected statistically significant sea level changes with positive and negative variable velocities and accelerations in sea level throughout their records, in addition to their secular trends and semi-annual, annual, and nodal periodicities.

The stations that exhibit variable accelerations indicate that, starting 1960s, almost all 13 stations out of 27 stations examined in this study, reveal increasing accelerations. However, it will be a leap of faith to infer that this is a global phenomenon, because, clearly, not all stations experience such variability. Yet, it is also not possible to rule out that the other stations do not experience variable accelerations with confidence because, all the *no-acceleration* model solutions (as well as *variable-acceleration models*) are not impressive enoughto eliminate this possibility.

Although all the model solutions pass F-tests for their predictive power, partly thanks to the large number of data, their adjusted R^2 values in Table 1 are clustered around 30 to 40 percent explained variation in the tide gauge series, which leaves room for detecting variable accelerations in the remaining stations' tide gauge data, that is if they indeed exist, in future studies with improved models.

Because of the prevalence of statistically significant variable accelerations, the velocity estimates of all the earlier studies using the same stations are *biased* as a result of these unmodeled effects. Moreover, their statistics, namely standard errors and reported adjusted R^2 values are also *biased* as a result of not modeling serial correlations in tide gauge records.

As a final note, the use of average velocity, given by the Eq. (9), is a must to properly calculate secular trends in the presence of either *constant*, *linear*, or *variable* accelerations.

Interestingly, *variable-accelerations* lend themselves equally well to another interpretation. Variable changes in the sea level, shown on the first panel of each station in Figure 1, can also be attributed to a number of hidden unmodeled periodic changes and their interaction, because they are small in magnitude and their periods exceeding decadal or longer periods that cannot be easily detected using spectral methods requiring records much longer than a century (unless their periods are known *a priori*). If that is the case, then the quartic model proposed in this study approximates their effects and enables unbiased estimation of secular trends. This duality between variable accelerations and unmodeled periodic sea level changes offer new challenges that are yet to be addressed.

Note that this is an introductory analysis with limited number of tide gauge stations emphasizing the issues related to the side effect of variable sea level accelerations at longer time scales on the sea level trends. The underlying dynamic interpretation of the kinematic models that were proposed in this study is needed. Use of average velocity in the presence of variable sea level velocity and accelerations is a must if polynomial models are deployed.

This study also recognized and properly modeled first order autoregressive disturbances in the tide gauge data at the global scale. Numerical results provide statistically significant evidence that they are all indeed important.

This study is by no mean exhaustive. The estimates are related to local relative sea level changes with limited number of stations and proper physical interpretation of the kinematic models all do require further studies for a global synthesis.

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References

Church J. and White N. J., 2006, A 20th century acceleration in global sea level rise, Geophys. Res. Lett., 33, L01602.

Douglas B. C., 1992, Global sea level acceleration, J. Geophys. Res., 97, 12, 699–12,706.

Douglas B. C., 1991, Global sea level rise. J. of Geophys. Res. 96(C4), 6981-6992.

Gornitz V., Lebedeff S. and Hansen J., 1982, Global sea level trend in the past century. Science (215) 1611-1614.

Holgate S. J. and Woodworth P. L., 2004, Evidence for enhanced coastal sea level rise during the 1990s, Geophys. Res. Lett. 31, L07305.

Holgate S. J., 2007, On the decadal rates of sea level change during the twentieth century, Geophys. Res. Lett. 34, L01602.

Houston J. R. and Dean R. G., 2011, Sea-Level acceleration based on U.S. tide gauges and extensions of previous global-gauge analyses, J. of Coastal Res., 27, 409 – 417.

Hildreth G. and Lu T., 1960, Demand relations with autocorrelated disturbances, Technical Bulletin 276, Michigan State University Agricultural Experiment.

IPCC, 2007 In Climate Change, 2007, The Physical Science Basis. Contribution of Working Group I to the Fourth Assessment Report of the Intergovernmental Panel on Climate Change. Solomon, S., D. Qin, M. Manning, Z. Chen, M. Marquis, K. B. Averyt, M. Tignor and H. L. Miller (eds.). Cambridge University Press.

Iz H. B. and Shum C. K., 2000, Mean sea level variations in the South China Sea from four decades of tidal records in Hong Kong, Marine Geodesy Vol.23, (4), 221-233.

Iz H. B. and Ng H. M., 2005, Are the global tide gauge data stationary in variance? Marine Geodesy, (3) 209-217.

Iz H. B., 2006, How do unmodeled systematic MSL variations affect long term sea level trend estimates from tide gauge data? J. of Geodesy, Vol. 80, No. 1, 40-46.

Iz H. B. and Ng H. M., 2011, Empirical modeling and impact of transient effects on the mean sea level trend estimates from the global tide gauge data, J. of Geodetic Sci., 1(3), 221-232.

Iz H. B., Berry L., Koch M., 2012, Modeling regional sea level rise using local tide gauge data, J. of Geodetic Sci., Vol. 2(3), 188–199.

Jevrejeva S., Moore J. C., Grinsted A., and Woodworth P. L., 2008, Recent global sea level acceleration started over 200 years ago? Geophys. Res. Lett. Vol.. 35, L08715, doi:10.1029/2008GL033611.

Maul G. A., Douglas M., 1993, Sea level rise at Key West, Florida, 1846-1992: America's longest instrument record? Geophy. Res. Lett., 20, 18, 1955-1958.

Neter J. M., Kutner H., Nachtsheim C. J., and Wasserman M., 1996, Applied linear statistical models, Richard D. Irwin, 1408.

Permanent Service for MSL, 2011, `http://www.psmsl.org/`, accessed on April 2011.

Runge, C., 1901, Über empirische funktionen und die interpolation zwischen äquidistanten ordinaten, *Zeitschrift für Mathematik und Physik* 46 224–243.

Woodworth P. L., 1990, A search for accelerations in records of European mean sea level. Int. J. Climatol., 10 129–143. doi: 10.1002/joc.3370100203.

Woodworth P. L., White, N. J., Jevrejeva, S., Holgate, S. J., Church, J. A. and Gehrels W. R., 2009, Evidence for the accelerations of sea level on multi-decade and century timescales. Int. J. Climatol., 29 777–789. doi: 10.1002/joc.1771.

Watson P. J., 2011, Is There evidence yet of acceleration in mean sea level rise around mainland Australia? J. of Coastal Res., 27, 368 – 377.