Title:	**Groundwater Drought Management by a Feedforward Control Scheme**
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Target Journal:	Water Resources Research

Drought management at the District has generally been based on the historical experiences. Groundwater levels are routinely monitored, and when critical levels are reached, action is taken through water use restrictions. When water levels recover, the restrictions are lifted. This paper presents a new modeling approach to make drought management more proactive.

A new control scheme was developed to manage regional groundwater drought conditions of the multi-layered aquifer system in Collier County, Florida. This scheme consists of a forecasting equation and a control equation based on the empirical relationship between head change and the corresponding pumpage/ recharge. After forecasting heads for the next month in the aquifer system and calculating the deviations of these heads from the target levels, the recommended spatial pumpage reduction rate is computed.

Simulation results, using several synthetic drought events having different frequencies, showed that the spatial variation of the estimated pumpage reduction was very significant, compared to the pumpage reduction variation caused by changing the previous drought return period. This result strongly supports the spatial forecasting and control of groundwater heads in the model area as proposed in this new control scheme.

It is, therefore, highly recommended that this type of groundwater drought management model be applied to other south Florida areas. Results of this model will be useful not only as a drought management tool during the anticipated drought periods, but also as a database for long-term historical groundwater heads which could be a valuable data source for water resources management.

DRE-321

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December, 1994

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Abstract

To manage regional groundwater drought problems at the multi-layered aquifer system in Collier County, Florida, a feedforward control scheme was developed. This scheme consists of a forecasting equation based on the Kalman filter algorithm associated with the space-time autoregression with exogenous variables model, and a control equation based on the empirical relationship between head change and the corresponding conceptual pumpage/ recharge term. The control mechanism is that, after forecasting one-month ahead heads in the multi-layered aquifer system and calculating the deviations from the 2-in-10-year target levels, the recommended spatial pumpage reduction rate is computed. Simulation results with several generated drought events having different frequencies showed that the estimated pumpage reduction was linearly proportional to the logarithm of return period of the antecedent drought, and that the spatial variation of the estimated pumpage reduction was greater than that of the frequency variation. The later result strongly supports the concept of spatial forecasting and control of groundwater heads in the model area as proposed in this control scheme.

1. Introduction

The water manager's goal of implementing a groundwater drought management plan is to protect groundwater resources and to assure equitable distribution of water to the different users during the anticipated drought condition so that adverse economic, social, and health impacts from the water shortage will be minimized. Since drought management plans should be based on the past, present, and future water conditions, forecasting of future droughts is inevitable. The purpose of forecasting in here is to control the groundwater system during the anticipated drought. Since forecasting in general is accompanied by considerable uncertainty, the following control problem should also consider the uncertainty. That is why many control problems have been handled by the stochastic time series topics. Along with stochastic forecasting theories, there exist a variety of stochastic control theories [*Box and Jenkins*, 1976, page 423; *Bennett*, 1979, page 533]. However, application of those theories to real groundwater system is much more complicate due to the space-time dependent nature of groundwater heads, but those complicate problems are rarely addressed in literatures. Thus, this paper intended to develop a state-of-the-art control method to handle the groundwater drought management problems.

As a methodology, a feedforward control scheme was adopted, which consists of a forecasting equation and a control equation. The forecasting equation was built by the Kalman filter algorithm associated with a state-space form of the space-time autoregression model with exogenous variables (STARX) model [*Ahn*, this issue]. The control equation, which estimates recommendation for reducing permitted groundwater use based on the anticipated deviation from the target water level, was developed by the empirical relationship between head change and a conceptual pumpage/recharge (PR) term. The proposed control scheme was tested in Collier County, Florida, with the generated drought conditions having different frequencies and the test statistics are presented at the end of this paper. The test results showed promising options for managing the groundwater resources during the anticipated drought periods.

2. Review of Existing Regulation Rules

Groundwater Use Permits

A definition of hydrologic drought given by *Dracup et al.* [1980] is "a water shortage with reference to a specified need for water in a conceptual supply and demand relationship." This definition indicates that drought is defined by a relative sense usually in terms of frequency of historical hydrologic events. Statistically, the probability of a drought event p, which is opposite to that of flood, is defined by the following non-exceedance cumulative density function F(.):

$$p = F_X(x) = prob.\{X \leq x\} \tag{1}$$

where X is the random variable denoting a hydrologic event such as rainfall depth or groundwater head, and x is any given reference number. Then, the return period Tr of any drought event is given by 1/p, and the complementary probability q is given by (1-p). For the definition of reference to a specified need for water, it is necessary to know the existing rules for groundwater uses and drought management in the model area. The model area referred in here consists of the west of Collier County and the southeast portion of Lee County, Florida.

The Florida Statutes (Part II of Chapter 373) states that the South Florida Water Management District (the District) is responsible for the permitting of the use of both surface and ground water within its jurisdictional boundary [*SFWMD*, 1993]. Without having any significant storage facilities, groundwater uses are the main water supply sources in the model area. *Bennett* [1992] estimated that agricultural and landscape irrigation withdrawals account for approximately 78 percent of the total groundwater use in the model area based on 1988 estimates. The permit information manual Volume III [*SFWMD*, 1993] specifies that the reasonable need for the irrigation water use is defined by the supplemental water requirement (SWR:s_t). To estimate the SWR for crops, this manual recommends to use the modified Blaney-Criddle equation for the evapotranspiration (ET: E_t) and the Soil Conservation Service (SCS) method for the effective rainfall (RE_t^*). That is, s_t(mm/month) is given by

$$s_t = E_t - RE_t^* \tag{2}$$

and the modified Blaney-Criddle equation [*SFWMD*,1993; *Jensen et al.*, 1990] for E_t is

$$E_t = .254 k_t\, k_c T_t p_t \tag{3}$$

where, k_t is the climatic coefficient related to the mean monthly air temperature (=$0.0173T_t$-0.314), k_c is the monthly factor reflecting the growth stage of the crop type, T_t is the mean monthly temperature at month t (°F), and p_t is the percent of daytime hours of the year at month t. The SCS method [*SFWMD*, 1993; *Jensen et al.*, 1990, page 67] to compute RE_t^* (mm/month) is

$$RE_t^* = f_s\, f_e\, (1.25 R_t^{0.8242} - 2.93) 10^{0.000955 E_t} \tag{4}$$

where f_s is the soil factor given by f_s=$0.53+0.0116d-8.94\times10^{-5}d^2+2.32\times10^{-7}d^3$ with d is the net depth of application(mm), f_e is the conversion factor from the mean monthly rainfall to the rainfall having a given frequency, and R_t is the total rainfall (mm/month) at month t. To obtain the groundwater allocation, SWR in each month is computed by the above method with a 2-in-10-year frequency rainfall (f_e=0.8-0.87), and the maximum monthly allocation is determined by choosing the month which has the largest SWR, and the total volume of SWR (L^3/T) is obtained by multiplying total irrigation area and dividing by irrigation efficiency.

An attempt was made to compute the historical SWR series by the above procedure with historical rainfall (f_e=1.0) and temperature series, and to relate with the corresponding historical head changes. However, the result was unsatisfactory, implying that the SWR method, particularly the RE_t^* by the SCS method, does not adequately simulate the actual SWR in the model area. Thus, a conceptual PR function was developed which maximizes the correlation between the historical head change and the corresponding PR series.

Water Shortage Plan

The water shortage plan attached to *SFWMD* [1993] provides consistent rules, principles, and restrictions that apply to groundwater users, facilitating the management and enforcement of droughts within the District's boundary. This plan also provides for variances from the water use permits. That is, if there is a possibility that there will not be sufficient water available within a source class to meet the anticipated demands, water managers issue a water shortage order to the users. This order is activated when a drought is foreseen and remains active as long as water restriction is in effect. Specifically, the water shortage plan outlines the drought contingency plan by stating that "the current data shall be compared to historical data to determine whether estimated present and anticipated available water supply .... will be insufficient to meet the estimated present and anticipated demand." This plan establishes the severity of the groundwater drought condition with the reduction of water uses as follows:

Water shortage (W/S) phase	Color code	% reduction in overall demand
I. Moderate W/S	Yellow	Less than 15%
II. Severe W/S	Orange	Less than 30%
III. Extreme W/S	Red	Less than 45%
IV. Critical W/S	Purple	Less than 60%

However, there exists no objective definition or tool to determine the above water shortage phases, and the simple frequency analysis of the current rainfall events and uniform water use reduction plan at the regional scale have been used.

3. Feedforward Control Scheme

System control serves to specify what system inputs are required to achieve given output levels. One of the common ways of forecasting and controlling the system is by the stochastic time series model as a represented system. In the system theory, there exist a variety of control schemes, but three basic forms prevail: open-loop control, closed-loop control, and feedforward control [*Bennett*, 1979, page 577]. In the **open-loop control** scheme, a control rule is preset on the basis of available experience. This scheme can be adjusted only infrequently and tends to allow unexpected fluctuations. The **closed-loop control** scheme, or so called **feedback** device, compares the system output and the specified target, and makes adjustments based on the deviation. The **feedforward control** scheme offers the advantages of detecting disturbances before they affect system operations and a control action is then initiated to compensate for potential deviations in the output. The District's past drought management practice can be classified as an open-loop control scheme.

In terms of stochastic time series framework, groundwater flow is governed by the endogenous variable (space-time groundwater flow itself) as well as several exogenous variables, such as rainfall, evapotranspiration, pumpage, seepage, regional groundwater flow, etc. Among these exogenous variables, pumpage is the only controllable variable in the sense of the system theory. However, the historical pumping records in the model area, specially those of agricultural pumpages, are not available. Thus, a conceptual PR function was developed, which is similar to SWR, but functionally more comprehensive since it includes the pumpages from public water supply wells. Then the PR function was related to the head change to control the groundwater head.

Since a set of stochastic forecasting models is available in the area [*Ahn*, this issue], it is possible to know the system disturbance in advance by forecasting. Taking into account for both forecasting mechanism and controllability of groundwater system, a feedforward control scheme was set up as shown in Figure 1. An output from the scheme is spatially controlled groundwater heads. System inputs include the most recently measured heads, specified seasonal target heads, water use permit information, and unmeasured disturbances which is not an artificial input but the source of disturbances other than the measured components [*Box and Jenkins*, 1976, page 424]. The system equation forecasts future groundwater heads in each aquifer layer by the Kalman filter algorithm associated with the STARX models. The main function of the control equation is to manipulate the system to meet the specified target level, which will be discussed in detail.

With $h_{t+1,i}^m$ as the forecasted head at t+1, site i, and layer m, and $H_{k+1,i}^m$ (k=1,...,12) as the seasonal target head at the corresponding month k, the deviation from target $d_{t+1,i}^m$ can be defined by

$$d_{t+,i}^m = H_{k+1,i}^m - h_{t+1,i}^m \,. \tag{5}$$

Positive $d_{t+1,i}^m$ means deficit of water that needs a control action. It was assumed that the above deviations from targets provide information on the reduced pumpage in the form of PR function $s_{P+,i}$ as

$$s_{P+,i} = function\{d_{P-l,i}^m, \ldots, d_{P,i}^m, d_{P+1,i}^m, \quad m=1,\ldots,L\} \tag{6}$$

where P is the current time, P+ indicates a time period from P to P+1 since the PR is a cumulative term during that period, L is the number of aquifer layers, and l is the backward time lag. Then, the corresponding percent reduction of pumpage $r_{P+,i}$ can be computed by

$$r_{P+,i} = \frac{s_{P+,i}}{s_i^T} \times 100 \quad (\%) \tag{7}$$

where s_i^T is the groundwater allocation at site i computed by the conceptual PR function with the 2-in-10-year rainfall.

4. Real Time Operation of the System Equation

As a forecasting tool for the future groundwater heads, the Kalman filtering approach associated with the STARX model was adopted. Having nx variates and nz covariates in space, the STARX model [*Ahn*, this issue] which describes the current state x_t(nx×1) in terms of the previous states $\{x_{t-1}, \ldots, x_{t-Nq}\}$ and the covariate $\{z_t, z_{t-1}, \ldots, z_{t-Nk}\}$ is given by

$$x_t = \sum_{i=1}^{Nq} D_i \circ \Lambda_i \, x_{t-i} + \sum_{j=0}^{Nk} E_j \circ \Omega_j \, z_{t-j} + w_t \tag{8}$$

where Nq and Nk are the temporal orders, Λ_i(nx×nx) and Ω_j (nx×nz) are the parameter matrices, D_i(nx×nx) and E_i(nx×nz) are the known spatial index matrix (SIM), w_t is a (nx×1) white noise vector having covariance Q, and notation (∘) is the Hadamard product. The m-th row and n-th column element $d_{i,mn}$ of D_i (same for the E_j) is 1 if m and n sites are i-th time lag neighbor, or 0 otherwise. The measurement equation can be expressed by

$$y_t = M(t)\, x(t) + v_t \tag{9}$$

where $x(t)' = [x_t', \ldots, x_{t-Nq}']$, y_t is a (ns×1) incompletely measured vector at time t with ns=Nq×nx, M(t) is the (nx×ns) measurement matrix, and the measurement noise v_t is a (nx×1) multi-Gaussian white noise having $v_t \approx N(0,R)$.

The Kalman filter forecasting requires a forecasting parameter set which includes the STARX model parameters, state and measurement noise covariances, and updated state and error covariances. Ideally, these parameters can be calibrated using the most currently measured data at each control step. However this option is impractical since the calibration process of a large scale model requires a great deal of time and efforts. Fortunately, the

Kalman filter algorithm allows updating the state and its associated error covariance terms continuously using the most currently measured data without recalibrating the entire system. Such an updating was defined as the **warm-up** process.

Figure 2 illustrates the concept of real time forecasting in the time horizon. The state estimator at each stage can be defined as follows. With notation of $\hat{x}_t^t$ as the estimated state of x(t), the conditional expectation of $\hat{x}_t^t$ is defined by

$$\hat{x}_t^s = E[x(t) \ |y_1,...,y_s, z_1,...,z_s] \tag{10}$$

where s is the span of the measurement. If the Kalman smoother is used at the calibration stage (s=T), the state estimator can be given by

$$\hat{x}_t^T = E[x(t) \ |y_1,...,y_T, z_1,...,z_T], \ \ for \ t=1,...,T. \tag{11}$$

At the warm-up stage, s is P(the present time) and the state estimator can be obtained by

$$\hat{x}_t^t = E[x(t) \ |y_1,...,y_t, z_1,...,z_t], \ \ for \ t=T+1,...,P \tag{12}$$

in which, complete or partial measurements of $\{y_{T+1}, ..., y_p\}$ should be provided. If a complete data set is available during the warm-up period, the state vector can be updated while the error covariances remain constant (time-invariant). At the forecasting stage (s=P), the l lead time forecasting with a set of complete-data during the warm-up stage is expressed by

$$\hat{x}_t^P = E[x(t) \ |y_1,...,y_P, z_1,...,z_P], \quad t=P+1,...,P+l \tag{13}$$

For examples, the one-step ahead forecasting is given by $\hat{x}_t^{t-1}$=E[x(t)|$y_1,...,y_P$ $z_1,...,z_P$], or if y_P is not measured at all, the two-step ahead forecasting is given by $\hat{x}_t^{t-2}$= E[x(t)|$y_1,...,y_{P-1}$ z_1, ...,z_P]. If warm-up period is too long, the state update may not be sufficient and the whole parameter set should be recalibrated. For the practical purpose of groundwater drought managements, it is recommended to calibrate forecasting parameters every other year, mainly during the off-drought period or non-cultivating season.

5. A Conceptual Pumpage/Recharge (PR) Function

The purpose of developing a PR function is to estimate the amount of pumpage requirement under the given climatic condition. Along with the feedforward control scheme, the PR function was based on the monthly time step with spatial resolution created by the Layer 1 gaging stations. Moreover, the boundaries of gaging stations were assumed to be defined by the Thiessen polygons. There exist a variety of micro-scale hydrologic theories, however it is not necessary to use such sophisticate hydrologic theories for the above space-time resolutions. Thus, the following simplified PR function was used: First, considering the SWR for crops (2) and the pumpage from public water supply wells, the PR function (mm/month) $s_{t,i}$ at time t and site i(=1,...,48) can be defined by

$$s_{t,i} = E^*_{t,i} - R^*_{t,i} + E_{pub}(i) \tag{14}$$

where $E^*_{t,i}$ is the average depth of water requirement for crops at polygon i, $R^*_{t,i}$ is the effective rainfall contributed to the groundwater system, and $E_{pub}(i)$ is the permitted public water supply converted to the equivalent depth of water (mm/month) at polygon i. Using the concept of infiltration capacity, $R^*_{t,i}$ can be expressed by

$$R^*_{t,i} = \begin{cases} R_{t,i} & \text{if } R_{t,i} \le R_{max,k} \\ R_{max,k} & \text{otherwise} \end{cases} \tag{15}$$

where $R_{t,i}$ is the measured monthly rainfall depth, and $R_{max,k}$ is the maximum rainfall depth contributing to the groundwater system at month k (k=1,...,12). In the study area, 18 rainfall stations were available (Figure 3), from which the Thiessen method was used to obtain $R_{t,i}$'s.

The Blaney-Criddle method in (3) indicates that the SWR for crops is obtained by the ET which is computed from the crop growth factor and soil type, both of which are spatial variables. Thus, a simplified form of $E^*_{t,i}$ (mm/month) can be expressed by

$$E^*_{t,i} = d_i \, k_c(k,i) \, E_k \tag{16}$$

where d_i is the net soil depth of application ranging from 10 mm to 90 mm [*SFWMD*, 1993], $k_c(k,i)$ is the factor reflecting the growth stage of a crop, and E_k is the monthly pan ET rate at month k. Although the daily temperature fluctuates significantly, the monthly mean temperatures are more or less stationary in space and time (Figure 4). This temperature stationarity justifies the use of the monthly mean ET in estimating $E^*_{t,i}$. There are a variety of agricultural practices in each thiessen polygon, but all agricultural types were classified into one of five types (Figure 5). With n_c different crops including a non-agricultural zone in polygon i, a composite crop factor within polygon i at month k, denoted by $k_c(k,i)$, can be computed by

$$k_c(k,i) = \sum_{j=1}^{n_c} w_j \, k_c(k,i,j) \tag{17}$$

where w_j is the areal weight for the particular crop type j, that is, $w_j = A_{ij}/A_i$ with A_{ij} is the area covered by the j-th crop type within the i-th polygon, and A_i is the total area at i. Table 1 lists k_c values for the distinct crop types.

For $E_{pub}(i)$ term, the following two assumptions were made: First, each public water supply well has a constant monthly pumping rate over year (non-seasonal). Second, the influence boundary of the drawdown caused by public water supply well is approximately defined by the 1.6 km buffer zone. Then, the $E_{pub}(i)$ can be given by

$$E_{pub}(i) = k_p \frac{Q_i}{A_i} \tag{18}$$

where $A_i(L^2)$ is the total area of the 1.6 km buffer zones of public water supply wells at polygon i, $Q_i(L^3)$ is the total public water supply allocation at polygon i, and k_p is the global factor for converting the Q_i to the equivalent depth of water . Figure 6 shows major public water supply wells with their 1.6 km buffer areas. Although the uniform buffer zone assumption for public water supply well is inaccurate, optimizing k_p by the conceptual PR function will compensate its weakness.

To summarize, the PR terms can be estimated by equations (14) through (18), with historical rainfall data, monthly average ET, spatial landuse and soil maps, public water supply well information (quantity and location), and an optimal PR parameter set $\{R_{max,k}, E_k,$ and k_p, $k=1,...,12\}$.

Optimizing the PR Parameter Set

The purpose of this optimization is to obtain a parameter set of the PR function which maximizes the correlation between the estimated PR term and the corresponding historical head change. The head change $\Delta h_{t,i}^m$, which is equivalent to the equation (5), was defined by

$$\Delta h_{t,i}^m = h_{t-1,i}^m - h_{t,i}^m \tag{19}$$

where $h_{t,i}^m$ is the groundwater head at time t, site i, and aquifer layer m. Both To measure the goodness-of-fit of the optimization, an average correlation coefficient between s_{t+i} and $\Delta h_{t,i}^1$ (only layer 1 because it is the most sensitive to the PR term) over space, $\rho_{\Delta h,s}$, were computed by

$$\rho_{\Delta h,s} = \frac{1}{nx}\sum_{i=1}^{nx} \rho_{\Delta h,s}(i) = \frac{1}{nx}\sum_{i=1}^{nx}\frac{cov[\Delta h_i\ s_i]}{\sigma_{\Delta h_i}\ \sigma_{s_i}} \tag{20}$$

where cov[.] and $\sigma_.$ are the sample covariance and standard deviation which were computed based on the historical groundwater heads and rainfalls (January 1977 - December 1993 with some missing data), s_i and Δh_i are the random variables of s_{t+i} and $\Delta h_{t,i}^1$ with t=1,...,T, respectively, and s and Δh are the corresponding spatial random variables. Then, the objective function was maximizing the $\rho_{\Delta h,s}$. The unconstrained nonlinear least square method [*IMSL*, 1991] with the finite-difference Jacobian method was used, whose results are listed in Table 2, where the estimated optimum objective function $\rho_{\Delta h,s}$ is 0.619.

6. Control Equation

If head change is explicitly modeled by the PR function, it is possible to build a control equation by combining it with the STARX model of head itself as described in *Box and Jenkins* [1976, page 424]. However, this approach causes difficulties in parameter calibration and equation handling since a multi-layered aquifer system creates a large state dimension. Instead, a separate control equation was developed based on the empirical relationship between the PR function and head changes in multi-layered aquifers. An alternative to this empirical control equation may be the physically-based groundwater flow models such as MODFLOW [*McDonald and Harbaugh*, 1988]. However, problems in such physically-based models are that model inputs, including rainfall and boundary conditions, should be forecasted in advance, that aquifer characteristics should be predefined, and that forecasting itself contains a lot of uncertainty so that the physically-based models does not increase accuracy proportional to the increased work load.

In order to make the system controllable, the PR function $s_{t+,i}$ was modeled by a time-lagged linear regression of head changes as

$$s_{t+,i} = \beta_{0,i} + \sum_{m=1}^{L} \sum_{j=0}^{Nk} \beta_{m,j,i} \, \Delta h^{m}_{t-j+1,i} + e_{t+1,i} \tag{21}$$

where Nk ($\geq$0) is the temporal model order, $\beta_{0,i}$ and $\beta_{m,j,i}$ are the regression parameters, and $e_{t,i}$ is the Gaussian white noise having a mean of zero and a variance of $\sigma_{e,i}$. This control equation does not use any of the spatial correlation structure, but as long as the forecasted heads $\Delta h_{t+1,i}$'s are forecasted via the STARX model, the estimated $s_{t+,i}$ based on $\Delta h_{t+1,i}$'s will more or less include the spatial correlations in it.

Nk in (21) can be determined similar to the identification process of stochastic time series models. As a test statistics for the model orders Nk=0,...,3, the spatial average AIC(Nk)'s [Refer to the original form in *Salas et al.* 1985, page 97] were computed by

$$AIC(Nk) = \frac{1}{nx} AIC_i(Nk) = \frac{T}{nx} \ln(\sigma^2_{e,i}) + 2Nk \tag{22}$$

where T is the sample size, and $\sigma_{e,i}^2$ is the residual variance. The period of record used in this identification procedure was from January 1987 to September 1993 (T=80). The computed AIC(Nk)'s are: AIC(0)=145.96, AIC(1)=122.17, AIC(2)=122.83, and AIC(3)=122.56, from which, Nk=1 was selected where the average R^2's is 0.657. For instance, the control equation for the 28-th polygon is

$$\begin{aligned} s_{t+,28} &= 41.0+7.293\Delta h^1_{t+1,28}+5.636\Delta h^2_{t+1,28}+1.997\Delta h^3_{t+1,28} \\ &\quad +2.896\Delta h^1_{t,28}+3.786\Delta h^2_{t,28}+2.005\Delta h^3_{t,28} \end{aligned} \tag{23}$$

with a R^2 of 0.765, where units of $s_{t,i}$ and $\Delta h_{t,i}{}^m$ are mm and meter, respectively. It should be noted that the superscripts in (23) are the layer indicators (not powers).

Similar to equation (10), the predicted pumpage reduction $\hat{s}^P_{P+,i}$ at present time P can be defined by the conditional expectation of

$$\begin{aligned} \hat{s}^P_{P+,i} &= E[s_{P+,i} \mid \Delta h^m_{t,i},\ t=1,\dots,P,\ m=1,\dots,L] \\ &= E[s_{P+,i} \mid (h^m_{t,i},\ t=1,\dots,p,\ m=1,\dots,L),\ \ (\hat{h}^m_{P+,i},\ m=1,\dots,L)]\ . \end{aligned} \tag{24}$$

Using the equation (21), the above expression can be written by

$$\hat{s}_{P+,i} = \beta_{0,i} + \sum_{m=1}^{L}\sum_{j=0}^{Nk}\beta_{m,j,i}\,\Delta h^m_{P-j+1,i}\ . \tag{25}$$

If the head measurement at time (P-1) is not available, a l-lead time pumpage reduction for t=P,...,P+l, can be obtained by

$$\hat{s}^P_{P+l,i} = E[s_{P+l,i} \mid (h^m_{t,i},\ t=1,\dots,p,\ m=1,\dots,L),\ \ (\hat{h}^m_{P+l,i},\ t=0,\dots,l,\ m=1,\dots,L)] \tag{26}$$

where the forecaster $\hat{h}^m_{P+l,i}$ is given by equation (13). In (26), if the system was not controlled at P-1, the l-lead time forecasted heads can be used to compute $\hat{s}^P_{P+l,i}$, or if the system was continuously controlled, the seasonal target heads should be used in (26).

7. Target Water Level

Since the agricultural pumpage from groundwater in the model area have been allocated based on the estimated SWR by the 2-in-10-year drought rainfall, the target water was set to the 2-in-10-year frequency heads. That is, any heads lower than that of 2-in-10-year frequency is subject to control. Further, it was assumed that the target water levels in each site are a monthly distributed along with the monthly SWR. To compute the monthly target heads, a two parameter normal distribution was fitted to the historical groundwater heads at each site and each month, from which the 2-in-10-year target head $H_{k,i}^{m}$ was computed using the 2-in-10-year frequency factor.

As constraints of these target heads, the following two criteria were used: If a site is located near the west coast (<8 km), any target head less than the mean sea level at Layer 1, 2, and 3 is set to 0 meter, in order to prevent salt water intrusion. Also, in the confined aquifer (Layer 4), any target head lower than the top elevation of the confined aquifer is set to the top aquifer elevation. This latter constraint is important to maintain the structural integrity of the limestone aquifer, in which the hydrodynamic pressure of the groundwater provides a significant amount of support against collapse and possible sinkhole formation.

The monthly 2-in-10-year PR terms were also computed by the proposed PR function with the 2-in-10-year monthly rainfall listed in Table 2, which is an monthly averages of the 27-year records in the region. Then, the allocation s_i^T in equation (7) at polygon i was determined by selecting the largest month (Figure 7).

8. Simulation and Discussion

The proposed feedforward control scheme was simulated for various conditions to investigate variations of pumpage reduction with respect to the different antecedent droughts. To know the seasonal variations, simulations were done for both the end of dry season (May) and the middle of wet season (September) cases with a 12-month warm-up period. For instance, if the present time P is in May, the corresponding warm-up period is from the previous June to May, and heads at June (P+1) are forecasted, from which the pumpage reduction (r_{P+i}) during June is computed. In each case, simulations were performed with six drought events whose return periods Tr are 2, 5, 10, 20, 50, and 100 years. Without having any historical drought after the model calibration, each drought events were artificially generated. That is, the groundwater heads in each month and each site having a given Tr were computed from the frequency factors determined by the fitted normal distributions prepared at the previous section.

The simulation procedure can be summarized as follows: After updating the state and error covariances during the warm-up period (t=P-11,...,P) by equation (12) using the generated heads, the next month heads (t=P+1) in each aquifer are forecasted by equation (13), from which the deviations from the seasonal targets are computed by equation (5). Finally, the pumpage reduction at each polygon is computed by equation (25), and the corresponding percent reduction is obtained by equation (7).

As summary statistics of the spatial pumpage reduction rates $r_{t+,i}$'s, i=1,...,48, the mean $\mu_r=E[r_{t+}]$ and standard deviation $\sigma_r=\{var[r_{t+}]\}^{1/2}$ were computed, where r_{t+} is an random variable with mass value $r_{t+,i}$, whose results associated with the return periods were plotted in

Figure 8. Figure 9 displays an example of contour map of the computed pumpage reduction rates for the dry season case with Tr=20 years. From the results of simulation, the following conclusions were drawn:

1. The estimated pumpage reduction was linearly proportional to ln(Tr) of the antecedent (warm-up period) head condition. This linearity was more acceptable when the groundwater head was less than the 2-in-10-year target level.

2. The spatial variation of the pumpage reduction was much greater than that of the order of return period (Figure 8b). This result strongly supports the concept of spatial forecasting and control of groundwater head in the model area, instead of the uniform pumpage reduction scheme previously practiced by the District.

3. The spatially averaged pumping reduction rate during the wet season was higher than that during the dry season, despite the higher expected rainfall during the wet season. This is mainly due to the increasing supplemental water requirement from the agricultural fields during the summer season. However, the spatial variance of reduction rates during the wet season was smaller than that of the dry season.

9. Conclusion

A feedforward control scheme was developed to manage regional groundwater drought problems in the multi-layered aquifer system located in Collier county, Florida. With input of the most currently measured heads, the control scheme provides a spatially varied pumpage reduction rates. The recommended pumpage reduction rate can be applied to the most of groundwater users in the model area since the calibrated P/R function accounts for the majority of groundwater uses including agricultural and public water supply pumpages. To test the proposed control scheme, simulations were performed for wet and dry seasons with different antecedent drought events, which exhibit the proficiency of the proposed control scheme. One of the inherent limitations of the proposed control scheme is that the recommended pumpage reduction is that of the lumped layers, not that of layer by layer operation, due to the adoption of the simplified control equation.

The main advantages of the proposed scheme are that the approach is proactive since the deviation from the target groundwater levels are predicted in advance, and that input as well as calibration are simple and thus more intuitive than that of the physically-based modeling approaches. Moreover this study demonstrates an application of the stochastic forecasting and control techniques to the large scale problems so that the proposed control scheme can be extended to the groundwater drought problems in the other area, as well as to the general scientific problems.

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List of Figures

Table 1. Monthly $k_c(k,i)$ values for different agricultural landuse types.

Landuse type	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
AC	0.63	0.73	0.86	0.99	1.08	1.13	1.22	1.06	0.99	0.91	0.78	0.64
AP	0.46	0.60	0.63	0.68	0.70	0.53	0.56	0.58	0.52	0.53	0.49	0.44
AM	0.63	0.66	0.68	0.70	0.71	0.71	0.71	0.71	0.70	0.68	0.67	0.64
AG	0.49	0.57	0.73	0.85	0.90	0.92	0.92	0.91	0.87	0.79	0.67	0.55

Landuse type: AC=cropland; AP=Pasture; AM=Groves, ornamentals, nurseries, tropical fruits; AG(AF and UO)=Grass [from *SFWMD*, 1993].

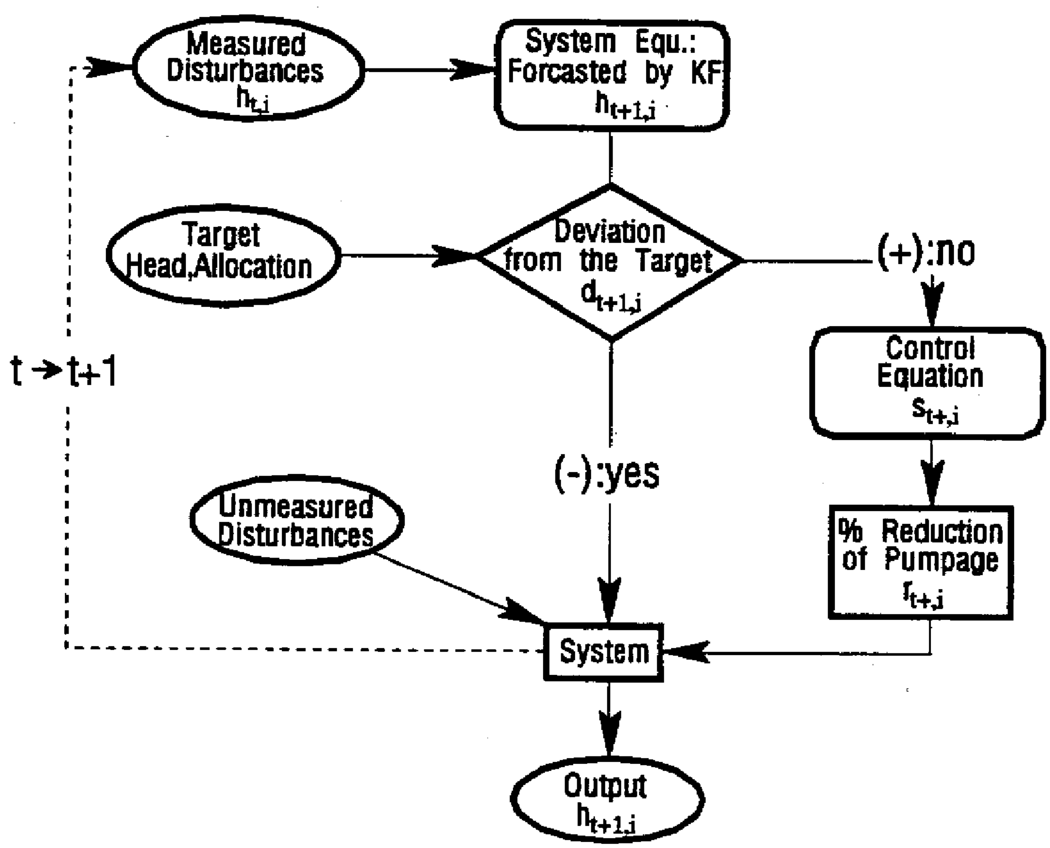
Table 2. Monthly mean rainfalls and optimal parameters for the PR function.

unit:mm

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
R_{mean}	42	99	103	64	108	194	189	191	222	104	30	37
$R_{max,k}$	76	152	140	152	152	191	241	140	114	216	76	279
E_k	114	135	135	132	74	50	132	193	203	241	180	132
k_p	0.832											

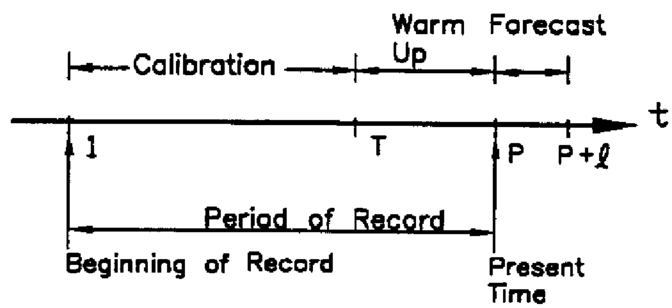
where R_{mean} is the historical monthly average rainfall, $R_{max,k}$ is the maximum rainfall contributed to the groundwater system, and E_k is the monthly pan ET depth, and k_p is the factor for converting the public water supply pumpage to the equivalent ET depth (unitless).

Figure 1



Measured
Disturbances
$h_{t,i}$
System Equ.:
Forcasted by KF
$h_{t+1,i}$
Target
Head,Allocation
Deviation
from the Target
$d_{t+1,i}$
(+):no
Control
Equation
$s_{t+,i}$
t → t+1
(-):yes
Unmeasured
Disturbances
% Reduction
of Pumpage
$r_{t+,i}$
System
Output
$h_{t+1,i}$

Figure 2


Warm Up
Forecast
Calibration
t
1
T
P
P+ℓ
Period of Record
Beginning of Record
Present Time

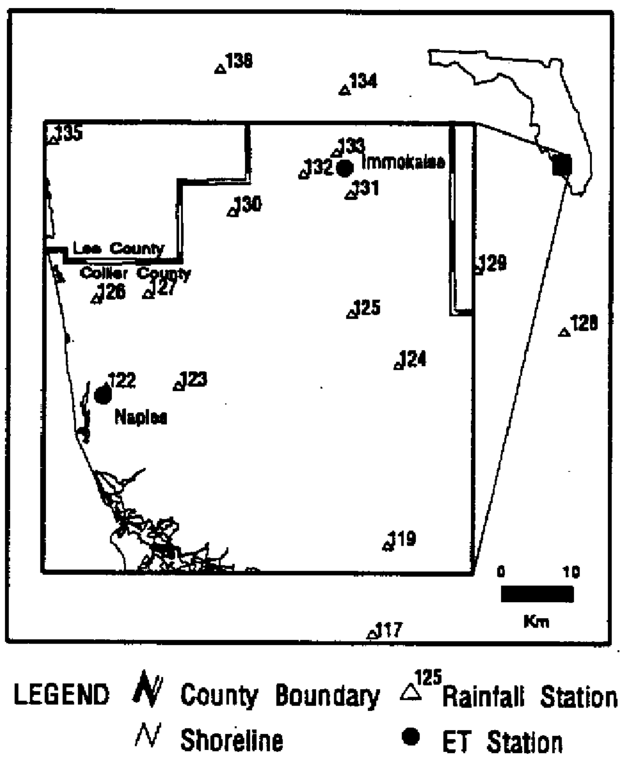
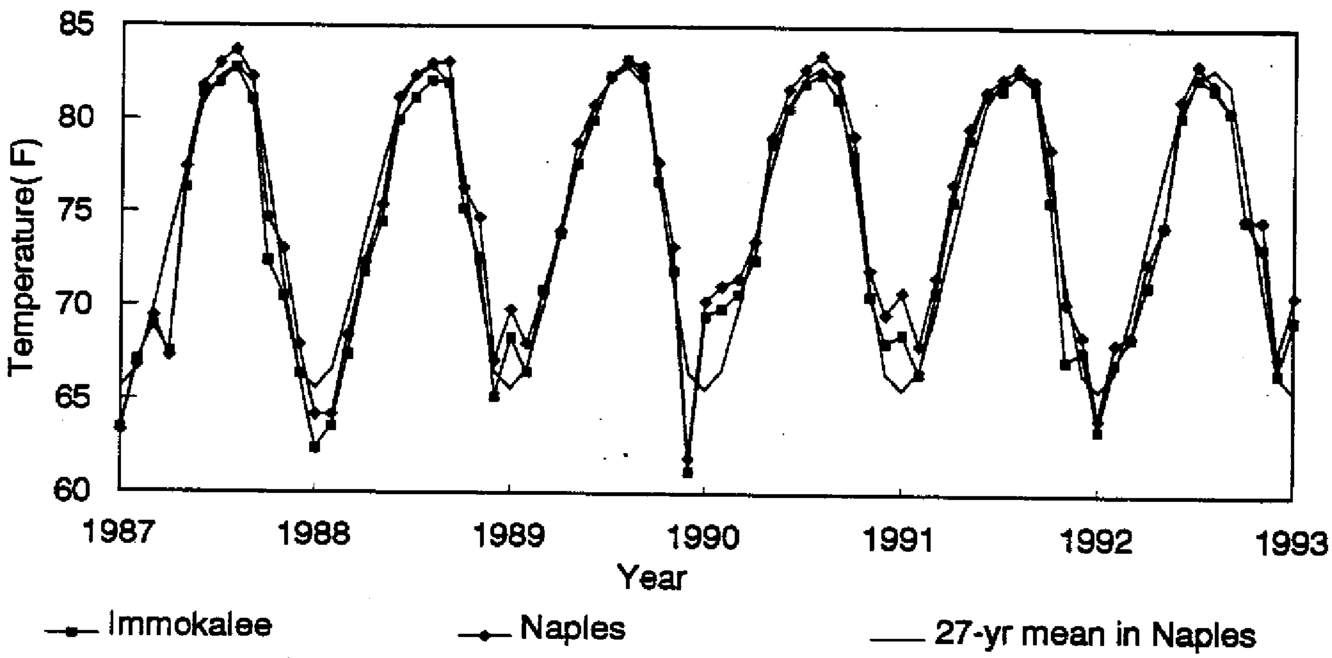
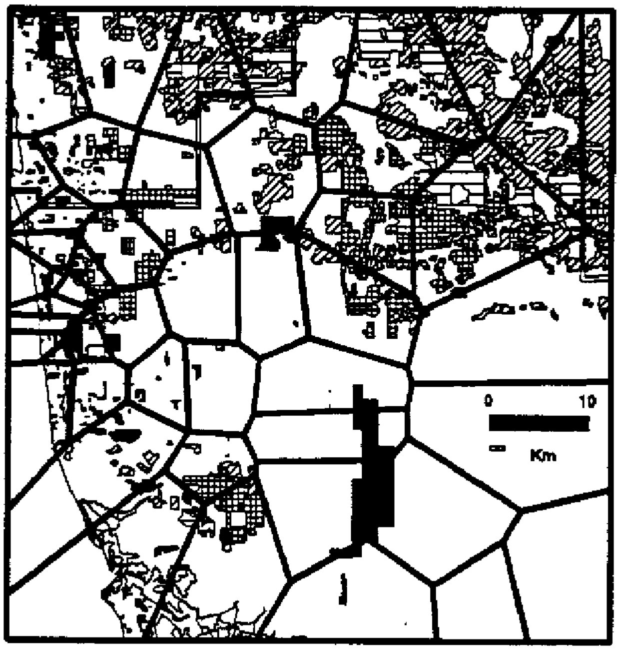

138
134
135
133
132
Immokalee
131
130
Lee County
Collier County
126
127
129
125
128
124
122
123
Naples
119
0
10
Km
117
LEGEND
County Boundary
125
Rainfall Station
Shoreline
ET Station

Figure 4


85
80
75
70
65
60
Temperature(F)
1987
1988
1989
1990
1991
1992
1993
Year
Immokalee
Naples
27-yr mean in Naples



AC (crop land)
AP (pasture)
AM (grove, nursery)
AF (dairy farm)
UO (golf, parks, rec. area)

LEGEND
County Boundary
Thiessen Polygon
Shoreline

Figure 6

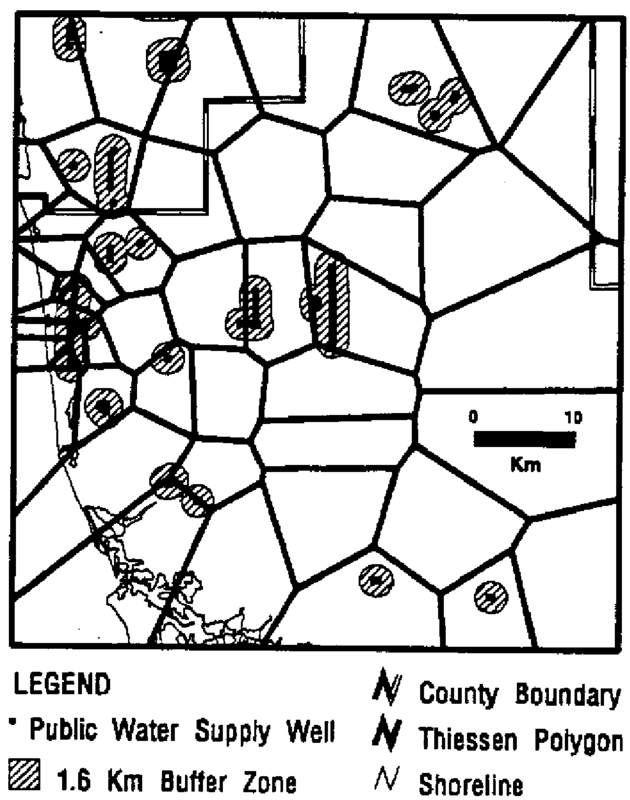
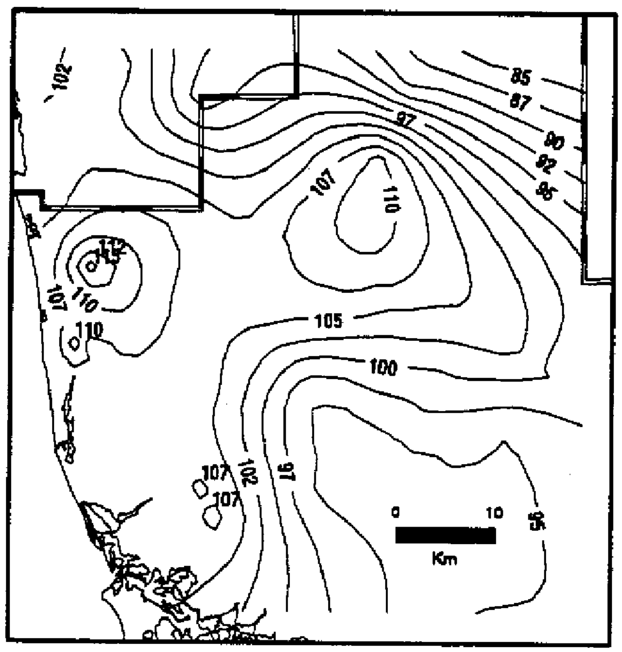

0
10
Km
LEGEND
Public Water Supply Well
1.6 Km Buffer Zone
County Boundary
Thiessen Polygon
Shoreline

Figure 7



LEGEND

County Boundary

Thiessen Polygon

Shoreline

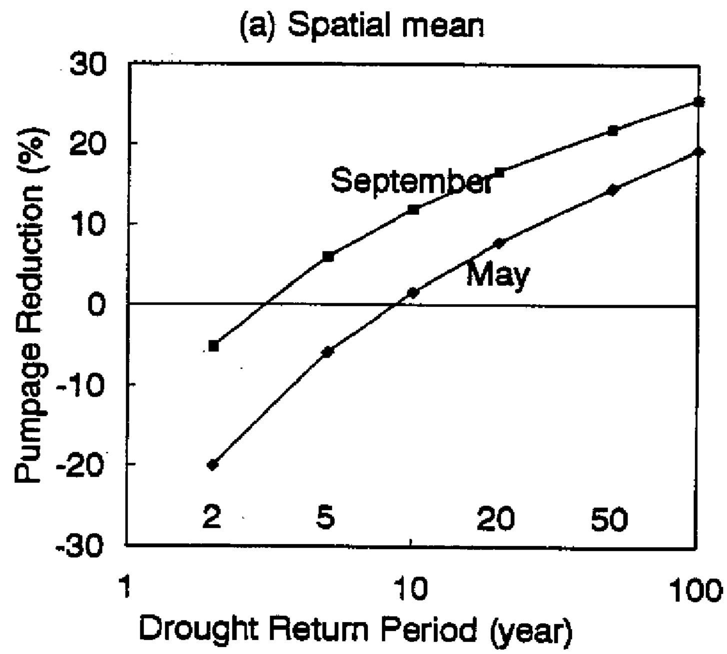
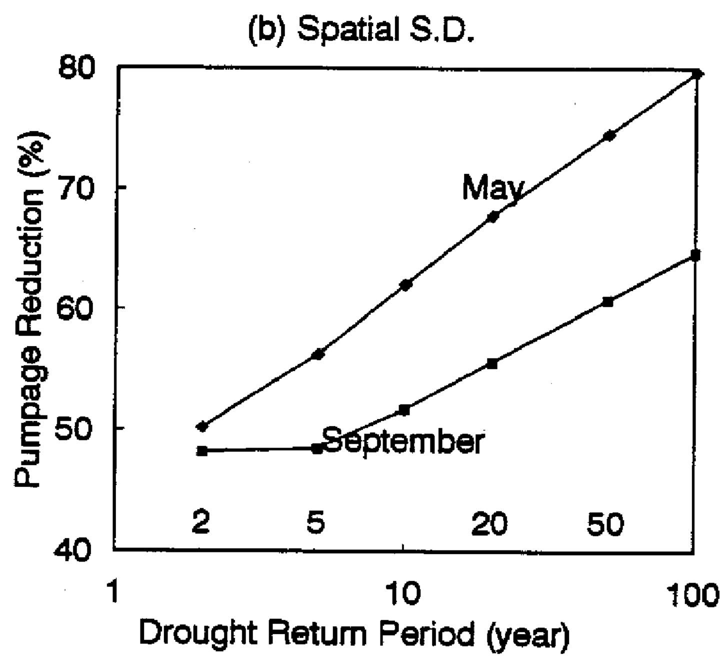
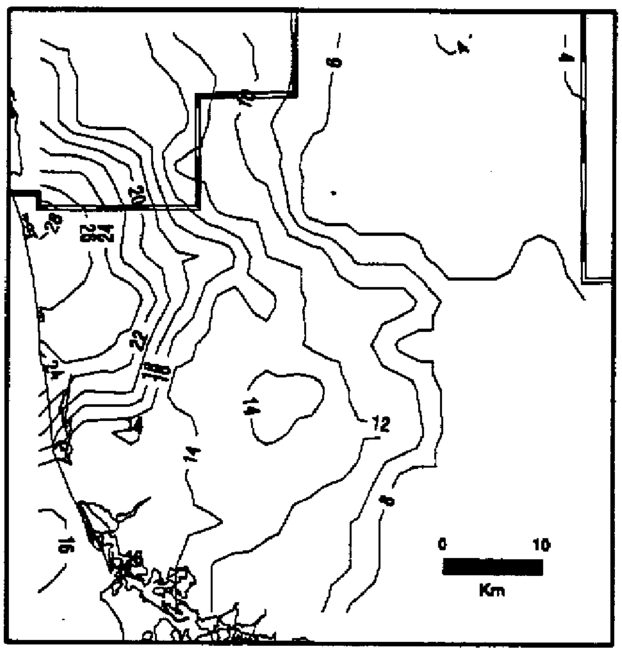

(a) Spatial mean
Pumpage Reduction (%)
30
20
10
0
-10
-20
-30
September
May
2
5
20
50
1
10
100
Drought Return Period (year)

Figure 8b



(b) Spatial S.D.
80
70
60
50
40
Pumpage Reduction (%)
May
September
2
5
20
50
1
10
100
Drought Return Period (year)



LEGEND

County Boundary

Thiessen Polygon

Shoreline