Technical Memorandum

EVALUATION OF RATING CURVES FOR EVERGLADES AGRICULTURAL AREA PUMP STATIONS S-5A, S-6, S-7 AND S-8, WITH FURTHER CALIBRATION OF RATING CURVES FOR S-8.

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CONTENTS

FIGURES

TABLES

ABSTRACT

Calibration of pump station rating curves is on-going at the South Florida Water Management District to increase the accuracy of flow calculations. A procedure to adjust heads and discharges collected at various pump-engine speeds is used to make data comparison at a given specific engine speed possible. An enhanced interpolation procedure between the two base rating curves used to compute the discharges is implemented, as well as the use of statistical criteria to determine the best match between experimental and predicted discharges. The calibration of the rating curves at pump station S-8 improved the accuracy of the predicted discharges. More discharge data needs to be collected to improve the certainty of flow prediction.

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EXECUTIVE SUMMARY

The South Florida Water Management District has among its goals to provide accurate flow data to the public. To ensure accuracy, the Data Management Division of the South Florida Management District conducts a flow data collection project to monitor the reliability of the data, which is calculated by a mathematical model called FLOW, developed at the District.

The present study analyzes the data collected at four District pump stations S-5A, S-6, S-7 and S-8 within the Everglades Agricultural Area. This analysis revealed that further calibration was needed for the rating curve at pump station S-8 only. The analysis uses statistical tools and experimental data to update or calibrate the flow equations (rating curve equations) used by FLOW. Such an analysis is necessary because most of the equations were developed in the 1960's, and do not account for possible errors affecting discharge measurements over the past 30 years.

For most of the pump stations of the Everglades Agricultural Area (EAA), there are two series of data available for calibration studies. The first series of data was collected between the late 1950's and the late 1960's; the new series was gathered from 1990 to present. Because both series are so far apart, they were compared against each other to find out whether they could be combined for calibration studies or not. A detailed description of the comparison procedure and the implementation of new rating curves are presented.

An example of data analysis conducted at pump station S-8 revealed that flow calculations with newly calibrated curves resulted in more accurate calculated discharges. The accuracy of these calibrated discharges can be further improved with the addition of data to be collected through the streamgaging project.

It is, therefore, recommended to continue the streamgaging effort and to extend the calibration methods presented in this report to other pump stations.

INTRODUCTION

Key elements in the mission of the South Florida Water Management District are flood protection, environmental protection and enhancement, water quality protection and water supply. To meet these objectives, reliable flow data are required. Customarily, the District calculates discharges using a computer program called FLOW which uses the rating curves of water control structures. The reliability of rating curves is critical to obtaining reliable discharge data.

A rating curve relates discharges to stage, headwater/tailwater differences for a given channel cross-section, gate opening, orifice size and pump speed. Most of the rating curves used in FLOW are either curves developed an provided by the pump manufacturers or curves developed by the United States Army Corps of Engineers using theoretical considerations or laboratory tests. The majority of these curves are not tested against in-situ experimental data. Although the theoretical approaches to compute flow are scientifically sound, they may not always match the measured discharges. Therefore, a calibration of the rating curves is necessary to obtain reliable flow data. "Reliable" is defined as an error of five percent or less.

To recalibrate the existing rating curves, a streamgaging program was initiated by the Data Management Division of the South Florida Management District (SFWMD) in 1990. The objective of this project is to collect flow data by gaging stream cross-sections close to pump stations, spillways, and culverts on a regular basis. The present streamgaging report deals with four of the Everglades Agricultural Area (EAA) structures, S5-A, S-6, S-7, and S-8 that discharge water into Water Conservation Areas 1, 2, and 3. These structures are pump stations which provide much of the inflow to Water Conservation Areas. Therefore, the focus of this study is on recalibration of pump station rating curves.

The key procedure in the calibration process consists of comparing the discharges obtained using FLOW to those measured at the pump stations from streamgaging. The comparison involves plotting the predicted data with the measured or experimental data. Statistical analysis of the differences in the data, such as the study of the goodness-of-fit of the predicted with the measured data, and the analysis of the relative errors between both discharges, helps make a decision whether to develop new rating curves by updating or correcting the equations used in the FLOW program (Turcotte and Mtundu, 1992). The details of the calibration procedures, the plotting of the data, the curve fitting process, the procedures describing the development of new rating curves, as well as a practical example of rating curve calibration at pump station S-8, are presented in this report.

The calibration procedure defined in this study can help calibrate pump performance rating curves and are applicable to other areas where verification of discharge calculation is desirable.

HISTORY OF RATING CURVES AT THE DISTRICT

Generally, factory pump performance rating curves are provided by the pump manufacturer. Factory curves are usually generated in the laboratory, through tests performed on hydraulic models. The results of the tests are then used to obtain the rating curves of the prototype through the laws of hydraulic similitude such as the dynamic and geometric similarities (Larsen and Padmanabhan, 1985; Beck, 1985).

The majority of performance rating curves at the District were developed by the U.S. Army Corps of Engineers (1962). However, there is no documentation indicating that those curves were tested against experimental data once the equipment (pumps and engines) was installed and operating. Each pump has an operation chart with a range of engine speeds at which it should be operated. The rating curves are developed for two different engine speeds. The rating curves in this study are based on engine speed instead of pump impeller speed because the latter is more difficult to obtain and because pump operation is traditionally reported as engine speed. There are two types of pump station rating curves: (1) the pump-mode rating curve and (2) the siphon-mode rating curve. The pump-mode curves are obtained by plotting discharge versus head when water is pumped from lower pool elevation (headwater) to higher pool elevation (tailwater), while siphon-mode curves describe the same type of plot when gravity is used to force water from a higher stage to a lower stage. The various rating curve equations and definition of variables are described in detail by Otero (1992). In this report, only the type 1 curves are investigated.

Pump-Mode Rating Curves

This type of rating curves exists in two forms; the first form represents the discharge, Q, as a third-order single-variable polynomial of the head H:

$$Q = C_0 + C_1 H + C_2 H^2 + C_3 H^3 \qquad (1)$$

in which:

Q = discharge rate

C_0, C_1, C_2, C_3 = regression coefficients

H = headwater/tailwater head difference

The second form represents Q as a two variable polynomial and can be written as follows:

$$Q = C_0 + C_1 X + C_2 Y + C_3 X^2 + C_4 XY + C_5 Y^2 + C_6 X^3 + C_7 YX^2 + C_8 XY^2 + C_9 Y^3 \qquad (2)$$

in which:

Q = discharge rate

X = head parameter, H/H_{fact}

in which:

H = headwater/tailwater head difference

H_{fact} = head factor

Y = pump speed parameter, $(\omega-\omega_{min})/\omega_{fact}$

in which:

ω = engine revolutions per minute

ω_{min} = a minimum engine speed below which the flow is assumed to be zero

ω_{fact} = engine speed factor

$C_0, C_1, C_2, C_3, C_4, C_5, C_6, C_7, C_8, C_9$ = regression coefficients.

This second equation is used for variable-speed pumps where the engine speed vary considerably during operations.

Discharge Calculation Procedure

Most of the equations used by the District to calculate discharges are single-variable polynomials. The flow is calculated by first choosing two base rating curves at fixed engine speeds (ω_1 [lower speed] and ω_2 [higher speed]), then interpolating or extrapolating between or outside those two curves. The final equation used to calculate the flow is defined as follows:

$$Q_P(H) = \left(\frac{\omega - \omega_1}{\omega_2 - \omega_1} \right) \left(Q_2(H) - Q_1(H) \right) + Q_1(H) \qquad (3)$$

in which:

$Q_P(H)$ = predicted discharge at engine speed ω for head H

$Q_1(H)$ = computed discharge at engine speed ω_1 for head H

$Q_2(H)$ = computed discharge at engine speed ω_2 for head H.

The required parameters are introduced in the equation of the curve and the discharges are directly calculated.

FACTORS THAT MAY AFFECT RATING CURVES CALIBRATION

For a complete calibration, all possible sources of error which can occur in the data collection process and in the development of original theoretical rating curves must be investigated. It will be impossible to come to acceptable conclusions in the calibration, if that process itself is carried out with erroneous data, and if one does not grasp the concepts behind the development of the original theoretical curves. The factors affecting calibration are:

(1) Rating curves shift due to an overall efficiency drop. When pumps at a given station are used over an extended period of time, they usually experience an efficiency drop because of the wear and tear on the system. This drop is often shown in the rating curve by a change of pattern (shift), generally due to the fact that the system gives lower discharges for a given head for the same power input to the system. On the other hand, when there is a replacement of pumps, the efficiency may improve. In either case, the rating curve may need to be adjusted to account for a shift.

(2) Changes in stream cross-section influence consistent accurate flow comparison. Over a long period of time, the gaged cross-section can change dramatically due to sedimentation, scour, aquatic plant growth or canal cross-section expansion or contraction by the SFWMD. In addition, data collectors may not gage the same cross-section from year to year because of lack of access to the location where previous discharge measurements were made. Such factors need to be examined closely to compare points collected at various time periods.

(3) Changes introduced by use of new equipment for flow data collection. The equipment used to collect data can change; the precision of those devices is not likely to be the same, so the data collected may not be comparable. Those changes need to be monitored in the data used for calibration.

(4) Human induced errors. Measurement errors can occur. The data collector may mistakenly report data or make errors in the flow computation. Such erroneous data need to be detected and removed from the calibration data, before any rating curve calibration is attempted.

(5) Changes may be introduced by construction of new hydraulic structures close to the gaging station. Occasionally, new structures, i.e., drainage or irrigation systems, are built on the canal and change the flow characteristics at the gaged cross-section. Those changes need to be taken into account when data are collected at different time periods.

(6) Measurement errors induced by changes of intake piping. If the intake from the headwater or tailwater is changed so that its opening is no longer perpendicular to flow direction, measured water levels will be influenced by water velocity in the vicinity of the intake. This process should be monitored by underwater inspection; if possible.

(7) Errors introduced in upstream stage readings by debris accumulation. Debris accumulation at the pump intake area may create pump headwater levels below what is measured by recorders in the pump-house, upstream of the debris. This process may introduce larger headlosses and lower than expected discharges. It must be monitored by installing water level measuring devices in the trash racks to allow a better evaluation of headlosses.

MATERIALS AND METHODS

DATA COLLECTION PROCEDURE

Since the rating curve calibration will not be effective if erroneous data are used, the emphasis is put not only on getting a large number of data points, but also on data quality.

The goal of the streamgaging team is to collect a minimum of three discharge values every week when pump stations are operating. The streamgaging data, the discharges stored in the database from periods prior to the streamgaging project, and the flow data obtained from the USGS are the main sources of data used in the calibration process in this study.

Streamgaging Flow Measurement Method

Discharges from the streamgaging program are obtained using the USGS streamgaging procedure described by Buchanan and Somers (1969). Other instruments and equipment used in the streamgaging process, as well as the detailed discharge calculation procedure, have been described by Buchanan and Somers (1969), Rantz and others (1982), and French (1985). A short description of the standard streamgaging procedure is presented later in this report. The streamgaging technique is the standard method used by the USGS to measure flow. It is reported (Rantz et al. 1982) that if single discharge measurements were made at a number of gaging sites using the streamgaging methods, the errors of two-thirds of the measured discharges would be less than 2.2 percent. This shows that the discharge measured through this method is very precise. The use of the flow data in calibration studies is therefore justified.

Parameters defining the flow conditions during the streamgaging process, such as headwater stages, tailwater stages, and the pump-engine speed, are also collected to plot the rating curves.

METHODS OF DATA ANALYSIS

PRELIMINARY DATA ANALYSIS

Relative Error Analysis for Four Everglades Agricultural Area (EAA) Structures (S-5A, S-6, S-7, and S-8).

A. General Remarks

The purpose of the following study was to determine, in light of the trends (in the data collected recently --1990-1991 period--) through streamgaging, which of four EAA structures, (S-5A, S-6, S-7 and S-8) needed immediate calibration. When it is determined that a structure needs calibration, the new data for the period 1990-1991, as well as prior data, will be investigated, as shown later in this report, to determine the possibility of combining both data sets for rating curve calibration analysis.

To meet the above objective, the mean relative error between discharges computed with the District FLOW program and the discharges measured through streamgauging were calculated. A zero mean relative error is an indicator of a good match between measured and calculated discharges. However, when there are relatively large positive and negative errors, the mean relative error (the average value of the individual errors) may fail to show those large departures between measured and calculated discharges. To correct that weakness in the analysis, the following criterion was proposed to determine structures with urgent need of calibration. The criterion is defined as follows:

1. the mean relative error must be zero at 5% level of confidence
2. 95 % of the relative errors must fall within ± 10 % of the flow rating curve

3. 100 % of the relative errors must fall within 15% of the rating curve, unless there is a reason not to do so (example: the data analyst is waiting to obtain future measurement to verify why a given discrepency is larger than 15%).

When the above three conditions are met, the rating curve of the interest does not need special attention, then a 95% confidence interval can be built around the mean relative error to show that the rating curve is reliable within that interval. When the conditions above are not met, the structure of interest is targeted for calibration.

1) Testing of Hypothesis on the Mean Relative Error. A t distribution defined by:

$$t = \frac{(X_{avg} - \mu)}{\frac{S}{N^{0.5}}} \qquad (4)$$

in which:

t = value of the t statistic

X_{avg} = mean of the sample of study

μ = true population mean

S = standard deviation of the sample of study

N = size of the sample of study,

was used (e.g., the t-distribution is suitable for statistical analysis when the sample size N is less than 30 and the population where the sample of size N is derived from, can be assumed to be normal) (Walpole and Myers, 1978). The hypotheses to be tested are:

H_{00}: $\mu = \mu_0 = 0$ i.e., the true population mean relative error is effectively zero, and

H_{11}: $\mu \neq \mu_0 = 0$ i.e., the true population mean relative error is significantly different from zero.

The test described by Walpole and Myers (1978) is as follows:

(1) compute the term

$$t = \frac{(X_{avg} - \mu_0)}{\frac{S}{N^{0.5}}} \qquad (\mu_0 = 0) \tag{7}$$

(2) obtain the $t_{\alpha/2,N-1}$ value from statistical tables

(3) compare t and $t_{\alpha/2,N-1}$. Reject H_{00} if $t > t_{\alpha/2,N-1}$ or $t < -t_{\alpha/2,N-1}$ otherwise accept H_{00} and conclude that the mean is not significantly different from zero.

2) Building a 95% Confidence Interval on the Mean Relative Error using the t Distribution. The lower limit (L) of the confidence interval is given by:

$$L = X_{avg} - S\frac{t_{\alpha/2,n-1}}{N^{0.5}} \tag{5}$$

and the upper limit (U) is defined by:

$$U = X_{avg} + S\frac{t_{\alpha/2,n-1}}{N^{0.5}} \tag{6}$$

in which:

L = lower limit of the confidence interval

U = upper limit of the confidence interval

X_{avg} = mean of sample of study

S = standard deviation of sample of study

N = size of sample of study

$t_{\alpha/2,N-1}$ = t value from statistical tables

α = level of confidence (Walpole and Myers, 1978)

N-1 = degrees of freedom for the statistical test

The 95% confidence interval is $I_{95\%} = (L, U)$.

B. Results of the Error Analysis for S-5A, S-6, S-7, and S-8

1) Hypothesis Testing. The data in Tables 1 2 3, and 4 were used to test whether the corresponding structures need calibration or not. The column showing the H_{00} status on Table 5 reveals the H_{00} hypothesis was accepted for all of the structures except for S-8. The mean error value is significantly different from zero at S-8 and not for S-5A, S-6 and S-7. However, when all three conditions of the test for calibration are considered, only S-5A does not need immediate calibration. S-6, S-7 and S-8 fail the last two conditions of not needing calibration (95% of the points do not have relative errors within 10% of the rating curve and 100% of data do not fall within 15% of the FLOW program rating curve for all three structures). The rating curves of all three structures need to be calibrated. However, since S-8 failed all three conditions of not needing calibration, it was the structure targeted for calibration in this report. The results of calibration procedures and curve fitting methods obtained for S-8 will be extended later to S-6 and S-7 but not in this report.

2) Confidence Interval. The procedure described above was applied to the flow data of pump station S-5A. The results are summarized in Table 5. The 95 percent confidence on the mean relative error value for that structure is $I_{95\%} = (L, U)$. The interpretation of the 95 percent confidence interval is that there is 95 percent confidence that the true mean value of the relative errors between measured and computed discharges lies within $I_{95\%}$. The confidence limits were reported for structures which did not pass the H_{00} test of hypothesis.

TABLE 1. Measured, Calculated Discharges and Relative Errors for S-5A.

Date	Operations (Engine Speeds) RPM	H Measured Head (ft)	Q_M Measured Discharges (cfs)	Q_P Discharges Computed With FLOW (cfs)	Relative Error $(Q_P - Q_M)/Q_M$
06/28/90	2 @ 700	5.60	1764	1670	-0.0530
06/24/91	4 @ 700	5.94	3013	3320	0.1019
07/03/90	3 @ 700	6.22	2295	2475	0.0784
07/23/91	3 @ 700	6.44	2445	2463	0.0074
07/29/91	4 @ 700	6.59	3227	3276	0.0152
10/24/90	3 @ 700	6.64	2481	2454	-0.0109
08/05/91	3 @ 700	6.66	2545	2454	-0.0358
07/29/91	4 @ 700	7.02	3225	3244	0.0059
10/24/90	3 @ 700	7.18	2183	2424	0.1104
09/23/90	4 @ 700	7.4	3104	3216	0.0361
N/A	N/A	N/A	N/A	Mean Relative Error	0.0255

TABLE 2. Measured, Calculated Discharges and Relative Errors for S-6.

Date	Operations (Engine Speeds) RPM	H Measured Head (ft)	Q_M Measured Discharges (cfs)	Q_P Discharges Computed With FLOW (cfs)	Relative Error $(Q_P - Q_M)/Q_M$
06/05/90	1 @ 500	2.50	1353	1011	-0.2528
01/17/91	1 @ 460; 2 @ 700	4.42	2821	2466	-0.1258
10/10/91	1 @ 500; 1 @ 700	4.57	1160	1538	0.3259
07/05/90	2 @ 500	4.81	2044	1880	-0.0802
06/10/91	3 @ 700	5.80	2751	2817	0.0240
07/11/91	3 @ 600	6.28	1623	1215	-0.2514
N/A	N/A	N/A	N/A	Mean Relative Error	-0.0601

TABLE 3. Measured, Calculated Discharges and Relative Errors for S-7

Date	Operations (Engine Speeds) RPM	H Measured Head (ft)	Q_M Measured Discharges (cfs)	Q_P Discharges Computed With FLOW (cfs)	Relative Error $(Q_P - Q_M)/Q_M$
07/17/90	1 @ 600	1.12	768	735	-0.0430
01/16/91	2 @ 650	1.42	1943	1772	-0.0880
10/12/90	1 @ 720	1.43	960	994	0.0354
10/10/90	1 @ 750	1.65	1021	1035	0.0137
01/17/91	1 @ 460; 2 @ 700	1.73	1927	2432	0.2621
01/17/91	1 @ 460; 2 @ 700	1.75	2012	2429	0.2073
06/19/91	2 @ 700	2.79	1928	1836	-0.0477
08/20/90	2 @ 720	3.19	1337	1870	0.3987
07/12/91	2 @ 720	3.66	1772	1830	0.0327
09/06/91	2 @ 720	3.98	1902	1800	-0.0536
09/10/91	2 @ 640	4.22	1538	1468	-0.0455
07/30/91	2 @ 720	4.74	1747	1720	0.0155
07/15/91	3 @ 720	5.08	2711	2487	-0.0826
N/A	N/A	N/A	N/A	Mean Relative Error	0.0441

TABLE 4. Measured, Calculated Discharges and Relative Errors for S-8

Date	Operations (Engine Speeds) RPM	H Measured Head (ft)	Q_M Measured Discharges (cfs)	Q_P Discharges Computed With FLOW (cfs)	Relative Error $(Q_P - Q_M)/Q_M$
07/16/90	1 @ 707	1.48	1077	1159	0.0761
07/06/90	1 @ 650	1.58	1019	1053	0.0334
07/16/90	1 @ 707	1.65	1068	1154	0.0805
07/06/90	1 @ 650	1.77	976	1045	0.0707
06/29/90	2 @ 600	2.40	1396	1854	0.3281
10/09/90	2 @ 680	2.47	2006	2146	0.0698
10/12/90	3 @ 700	2.96	3243	3273	0.0093
07/17/91	3 @ 700	3.16	2045	3252	0.0601
08/16/90	2 @ 650	3.32	1682	1960	0.1653
07/27/91	2 @ 700	3.67	1039	1062	0.0221
09/19/91	2 @ 580	3.70	1537	1648	0.0722
09/04/91	2 @ 680	3.74	2018	2040	0.0109
N/A	N/A	N/A	N/A	Mean Relative Error	0.0832

TABLE 5. Results of Mean Relative Errors Analysis at the EAA Pump Stations

Structure	Number of Data Points	Standard Deviation and Sample Average Error		Lower and Upper 95% Confidence Limits		t-Statistics Parameters		Status of H_{00} Test
	N	S	X_{avg}	L	U	T	$t_{0.025,N-1}$	
S-5A	10	0.163	0.053	-0.012	0.063	1.521	2.262	Accepted
S-6	6	0.198	-0.060	N/A	N/A	-0.745	2.571	Accepted
S-7	13	0.145	0.044	N/A	N/A	1.100	2.179	Accepted
S-8	12	0.084	0.083	N/A	N/A	3.430	2.201	Rejected

DESCRIPTION OF CALIBRATION PROCEDURE

Calibration of Base Rating Curves

A. General Procedure

Because flow computations are performed by interpolation between or extrapolation outside the base rating curves, the first step in the calibration procedure is to calibrate those two curves. The following sections describe the calibration process.

1. Adjustment of Measured Heads and Discharges. The heads and discharges collected during streamgaging trips to a pump station are obtained at various pump engine speeds. Therefore, they cannot be used to plot a rating curve at a given speed without adjustments made to the data. The following equations:

$$Q_2 = Q_1\left(\frac{\omega_2}{\omega_1}\right) \qquad (8)$$

in which:

Q_2 = adjusted discharge at speed of interest ω_2

Q_1 = measured discharge at speed ω_1

and

$$H_2 = H_1\left(\frac{\omega_2}{\omega_1}\right)^2 \qquad (9)$$

in which:

H_2= adjusted head at speed of interest ω_2

H_1 = measured head at speed ω_1,

proposed by Novak et al. (1989) and Karassik et al. (1985), were used to write a short computer program to compute the corrected discharge and head values at a given engine speed of interest. When the data available have enough measured heads and discharge values for each of the base engine speeds as judged by the data analyst, an adjustment of those data is not necessary. The data analyst can develop the rating curves by directly using the measured data available for each of the engine speeds. However, a sufficient an amount of data is not presently available for the structures investigated in this document. That is why the data adjustment procedure presented here is used before rating curves are developed.

2. **Calculation of the Predicted Discharge.** The adjusted heads are introduced in FLOW to calculate the theoretical rating curves at speeds ω_1 and ω_2. The speeds ω_1 and ω_2 are the lower and upper pump engine speeds for the normal range of operation of the pumps.

3. **Plot of Theoretical Rating Curve and Measured Data.** The theoretical rating curve is plotted with the measured data using a spreadsheet. Head (H) and discharge (Q) are the variables used for the plot, and they must be checked on the plot to eliminate outliers. These outliers may be errors of measurements incorporated in the data, or data that have been erroneously reported by the data collectors.

4. **Define Calibration Criterion.** An objective calibration criterion aims at giving the best match between discharges computed with the rating curve equations and the experimental discharges. The USGS criterion requires that every random departure (relative error between measured and predicted discharges) of a discharge measurement from its corresponding value, indicated by

the existing rating curve be less or equal to five percent for it to be a good check of that curve. Values outside that range are not acceptable (Rantz et al., 1982).

The criterion also states that when several values meet the five percent criterion and are plotted on one side of the existing rating curve, there is a shift towards that side of the rating curve.

As defined, the USGS criterion implies that the original rating curve should be very reliable. That is not necessarily the case for rating curves which have not been recently checked after the 1960's with site measured data, as are most of the rating curves studied in this report. The following procedure, derived from the USGS criterion, was used as a calibration criterion in this study: (1) data on the original rating curve where computed and experimental discharges meet the USGS criterion are considered good and are automatically kept; (2) data which present a departure larger than five percent from the existing rating curve are not categorically rejected; they are combined with data meeting the five percent criterion to obtain new curve by curve fitting methods; this approach gives more weight to the recent experimental data; (3) the coefficient of determination (correlation coefficient squared) R^2 between computed and experimental discharges should be between 0.6 and 1.0 (the lower bound of 0.6 on the coefficient of determination will be changed to a higher value as data become available); (4) more than a predefined percentage of the data points must meet the USGS five percent criterion; (5) points of the plot which were not rejected because there was not enough information to do so will be checked with future points collected by the on-going streamgaging project.

The USGS criterion is not the only criterion of good calibration. The USGS criterion can be combined with other criteria to impose desired constraints for goodness of fit in the calibration process. For that purpose, a summary of some other calibration methods is given, and described in the following paragraphs.

4.1 The Efficiency Coefficient Criterion (Haan et al., 1982). This criterion consists of defining the efficiency coefficient E of the calibration by the following equation:

$$E = 1 - \frac{\sum_{j=1}^{N} (Q_{Xj} - Q_{Mj})^2}{\sum_{j=1}^{N} (Q_{Xj} - Q_{XBAR})^2} \qquad (10)$$

in which:

N = number of data points in the sample of study

E = efficiency coefficient

Q_{Xj} = measured or adjusted measured discharge

Q_{Mj} = computed discharge

Q_{XBAR} = average of all experimental discharges

E is then compared to a oreviously chosen efficiency coefficient E_0. A value of E_0, close to but not large than unity, is desirable for a good calibration. If $E>E_0$, the calibration is said to be efficient. Otherwise, the calibration is not good.

4.2 The Least Squares Method. This method consists of performing a linear regression analysis between measured and predicted data. A coefficient of determination between 0.6 and 1.0 (for the flow data presently available) can indicate an acceptable agreement between experimental and predicted discharges.

4.3 The Chi -Square Method. The chi-square technique is a statistical tool used to verify the goodness of fit of an equation to a set of measured data points. In its simplest form, it can be defined as:

$$X^2 = \sum_{j=1}^{N} \frac{(Q_{Xj} - Q_{Pj})^2}{Q_{Pj}} \tag{11}$$

in which:

X^2 = chi-square statistic

N = number of data points

Q_{Pj} = predicted discharge

Q_{Xj} = experimental discharge

The statistic is then compared to a reference value obtained from statistical tables. If the computed statistic is less than the reference value, the fit is good, otherwise the fit is not good. A short computer program was written to allow automated calibration.

B. Curve Fitting

General Remarks. From documents provided by the U.S. Army Corps of Engineers [COE] (1962), the rating curves of most of the centrifugal pumps with axial-flow impeller, installed at South Florida Water Management District pump stations are of the rising characteristic type with downward concavity; i.e, the discharge decreases when the head increases. The downward concavity condition requires that the second derivative of the rating curve equation be negative and the rising characteristic condition imposes that the first derivative of the curve be non-zero over its applicable range; i.e., the rating curve should not have another maximum within its range of operation except perhaps the lower bound of the range of heads.

A preliminary study revealed that a function of the form:

$$Q = a_0 H^r \tag{12}$$

in which:

Q = discharge

H = head

a_0, r = real constants,

which require a logarithmic transformation before they can be fitted to the data, could not give a suitable result, because they could not meet the downward concavity condition (negative second derivative condition).

The above remark, coupled with reports by Brater and King (1976) and Novak et al. (1989) indicates that a pump characteristic rating curve can be described by a second-order polynomial of the form:

$$Q = aH^2 + bH + c \tag{13}$$

in which:

Q = discharge

H = head

a,b,c = constants,

led to the choice of the second-order polynomial as the equation to be fitted to the data to obtain single-variable polynomial pump rating curves.

The quadratic equation is considered good when it meets the conditions defined earlier. Those conditions, in the case of a quadratic equation, become:

$$Q''(H) = 2a < 0 \tag{14}$$

$$Q'(H) = 2aH + b \neq 0 \tag{15}$$

in which:

$Q''(H)$ = second derivative of Q with respect to H (equation 13)

$Q'(H)$ = first derivative of Q with respect to H (equation 13)

a,b = real constants of equation 13,

Downward concavity and rising characteristic conditions are met by equations 14 and 15, respectively. In addition to the conditions mentioned above, the quadratic equation should also meet the goodness-of-fit criteria previously defined in this paper for it to be accepted as a viable rating curve equation.

Curve Fitting Procedure.

1. Perform a direct fit of a second order polynomial to the data using the least squares method.

2. If the fitted equation meets the first and second derivative conditions, and if the coefficient of determination R^2 between calculated and experimental discharges is between 0.6 and 1.0, and if the departure (relative error) of the new curve from the old rating curve meet the USGS five percent relative error criterion, then use the new fitted curve as the new rating curve polynomial. In case the relative error condition or the R^2 condition cannot be strictly met because of shortage of data, the analysts may select a reference R^2 and a certain percentage of data which passes the USGS five percent relative error test to obtain a rating curve and use those new parameters to define the curve fitting criteria. In any event, the effort in selecting the criteria should be geared toward obtaining the best possible match between calculated and experimental discharges.

3. When the direct fit of the data as defined in step 1 does not meet the goodness-of-fit criteria, an attempt to use new curve fitting methods to obtain a rating curve polynomial should be made. In any case, the goodness-of-fit criteria should remain the same as described in step 2. Some of the curve fitting methods attempted in this present study are listed below.

3.1 The "Eyeball Fitted Curve" Method. This method consists of drawing a best-fit eyeball curve with the experimental data and using the points on that curve to fit a polynomial to the data using the least square method. If the new curve meets the conditions defined in step 2, it can be chosen as the rating curve polynomial. This method, proposed by Kennedy (1984), involved the judgement of the data analyst and can give good results when done by an experienced data analyst. Its major drawbacks are that it is a non-reproducible method and that it can introduce analyst-dependent bias in the analysis of the data.

3.2 The Reduced Least Squares Method. This method was developed during the course of this study and consists of finding the coefficients a,b,c of the quadratic equation 13 by requiring that the equation of the new curve satisfy the following constraints: (1) the rating curve must match a point P of coordinate H_0 and Q_0 of the original rating curve in a chosen area where no experimental data are available; (2) the sum of squared differences between the calculated and the experimental discharges must be minimized. The following procedure can be used to obtain the coefficients of the polynomial. The coefficient c can be obtained by substituting Q_0 and H_0 in equation 13:

$$c = Q_0 - aH_0^2 - bH_0 \tag{16}$$

in which:

c = real constant of equation 13,

Q_0 = discharge at head H_0

H_0 = head at intersecting point on existing curve

a,b = other coefficients of equation 13

the expression for c can then be substituted in equation 13 to give the following equation:

$$Q = a\,(H^2 - H_0^2) + b\,(H - H_0) + Q_0 \qquad (17)$$

in which:

Q = discharge

Q_0 = discharge at given head H_0

H_0 = head

a,b = other coefficients of equation 13.

Constraint (2) can then be used to obtain a and b by using the least squares method to minimize the expression:

$$SSE = \sum_{i=1}^{N} (Q_i - a\,(H_i^2 - H_0^2) - b\,(H_i - H_0) - Q_0)^2 \qquad (18)$$

in which:

SSE = sum of squared errors

Q_i = discharge for measurement i

H_i = head for measurements

a,b,H_0 and Q_0 are the same as defined in the previous equations.

Minimizing the term SSE using the least squares method, as described by Holman and Gajda (1978), implies that the partial derivatives of SSE with respect to a and b are equal to zero; i.e.,

$$\frac{\partial SSE}{\partial a} = 0 \qquad (19)$$

and

$$\frac{\partial SSE}{\partial b} = 0 \qquad (20)$$

By expanding equations 19 and 20, the following system of equations is obtained:

$$a\sum_{i=1}^{N}(H_i^2 - H_0^2)(H_i - H_0) + b\sum_{i=1}^{N}(H_i - H_0)^2 = \sum_{i=1}^{N}(Q_i - Q_0)(H_i - H_0)$$

$$a\sum_{i=1}^{N}(H_i^2 - H_0^2)^2 + b\sum_{i=1}^{N}(H_i - H_0)(H_i^2 - H_0^2) = \sum_{i=1}^{N}(Q_i - Q_0)(H_i^2 - H_0^2) \qquad (21)$$

The unknowns a and b are found by solving this system of equations. The third constant c is calculated by substituting a,b into equation 16.

The equation, thus determined, can be verified against the criteria of step 2; if all conditions are met, the new curve can be accepted as the new rating curve polynomial.

3.3 The Modified Least Squares Method. This method consists in finding the coefficients a, b and c of a quadratic equation by replacing one of the three equations of the least squares system of equations by the following equation:

$$Q_0 = aH_0^2 + bH_0 + c \qquad (22)$$

in which:

Q_0 = discharge of the matching point between calculated and estimated rating curve

H_0 = head at matching point

a,b,c = coefficients of the quadratic equation

Equation 22 is obtained by the requirement that the curve passes through a match point between predicted and calculated rating curves. The replacement of one of the least squares equations by Equation 22 yields three different systems of equations which need to be solved simultaneously to obtain the values of a,b,c (i.e., the values of these coefficients which meet the conditions defined in step 2).

The three systems of equations which need to be solved are listed as follows:

System 1

$$aH_0^2 + bH_0 + c = Q_0$$

$$a\sum_{i=1}^{N} H_i^3 + b\sum_{i=1}^{N} H_i^2 + c\sum_{i=1}^{N} H_i = \sum_{i=1}^{N} H_i Q_i \qquad (23)$$

$$a\sum_{i=1}^{N} H_i^2 + b\sum_{i=1}^{N} H_i + cN = \sum_{i=1}^{N} Q_i$$

System 2

$$a\sum_{i=1}^{N} H_i^4 + b\sum_{i=1}^{N} H_i^3 + \sum_{i=1}^{N} H_i^2 = \sum_{i=1}^{N} H_i^2 Q_i$$

$$aH_0^2 + bH_0 + c = Q_0 \qquad (24)$$

$$a\sum_{i=1}^{N} H_i^2 + b\sum_{i=1}^{N} H_i + cN = \sum_{i=1}^{N} Q_i$$

System 3

$$a\sum_{i=1}^{N} H_i^4 + b\sum_{i=1}^{N} H_i^3 + c\sum_{i=1}^{N} H_i^2 = \sum_{i=1}^{N} H_i^2 Q_i$$

$$a\sum_{i=1}^{N} H_i^3 + b\sum_{i=1}^{N} H_i^2 + c\sum_{i=1}^{N} H_i = \sum_{i=1}^{N} H_i Q_i \tag{25}$$

$$aH_0^2 + bH_0 + c = Q_0$$

in which:

Q_i = discharge at head H_i

H_i = head

Q_0 = discharge at matching point between calculated and estimated rating curves

H_0 = head at matching point

N = number of data points in the sample

a, b, c = constants in the quadratic equation.

This method can generally be used to obtain a rating curve polynomial when it is necessary to match a point on the original rating curve to a point on the new curve to determine the coefficients of that computed curve. This method appears to be better than the eyeball-fitted method because it involves less empirical bias and is based upon statistical considerations. The goodness-of-fit depends on the location of the matching point. Therefore, this method must be tried with several matching points until convergence is achieved.

C. Flow Calculation with the Calibrated Base Rating Curves

The ultimate goal of the calibration process is to use the calibrated base rating curves to predict flow for various engine speeds with at least 95 percent accuracy. An equation similar to Equation 3 was developed to calculate the predicted discharges. The procedure underlying the development of the equation is explained below. In Equation 3, the same head (H_R) for a given speed (see Figure 1) was introduced in the two base rating curve equations to obtain Q_1 and Q_2 and then interpolation was made between those two values to obtain Q_P, the estimated discharge. After a study of the interpolation procedure of FLOW, resulting discharges are closer to the measured discharges, when the adjusted heads H_1 and H_2 were used in the base rating polynomials instead of the original head H_R. That procedure shortens the interpolation range (see Figure 1). The flow calculation equation becomes:

$$Q_P(H_R) = \left(\frac{\omega - \omega_1}{\omega_2 - \omega 1}\right)\left(Q_2(H_2) - Q_1(H_1)\right) + Q_1(H_1) \tag{26}$$

in which:

$Q_P(H_R)$ = discharge at engine speed RPM at reference head H_R

H_1 = adjusted value of head H_R associated to speed ω_1

H_2 = adjusted value of head H associated to speed ω_2

$Q_1(H_1)$ = discharge calculated with lower speed (ω_1) rating curve polynomial

$Q_2(H_2)$ = discharge calculated with higher speed (ω_2) rating curve polynomial.

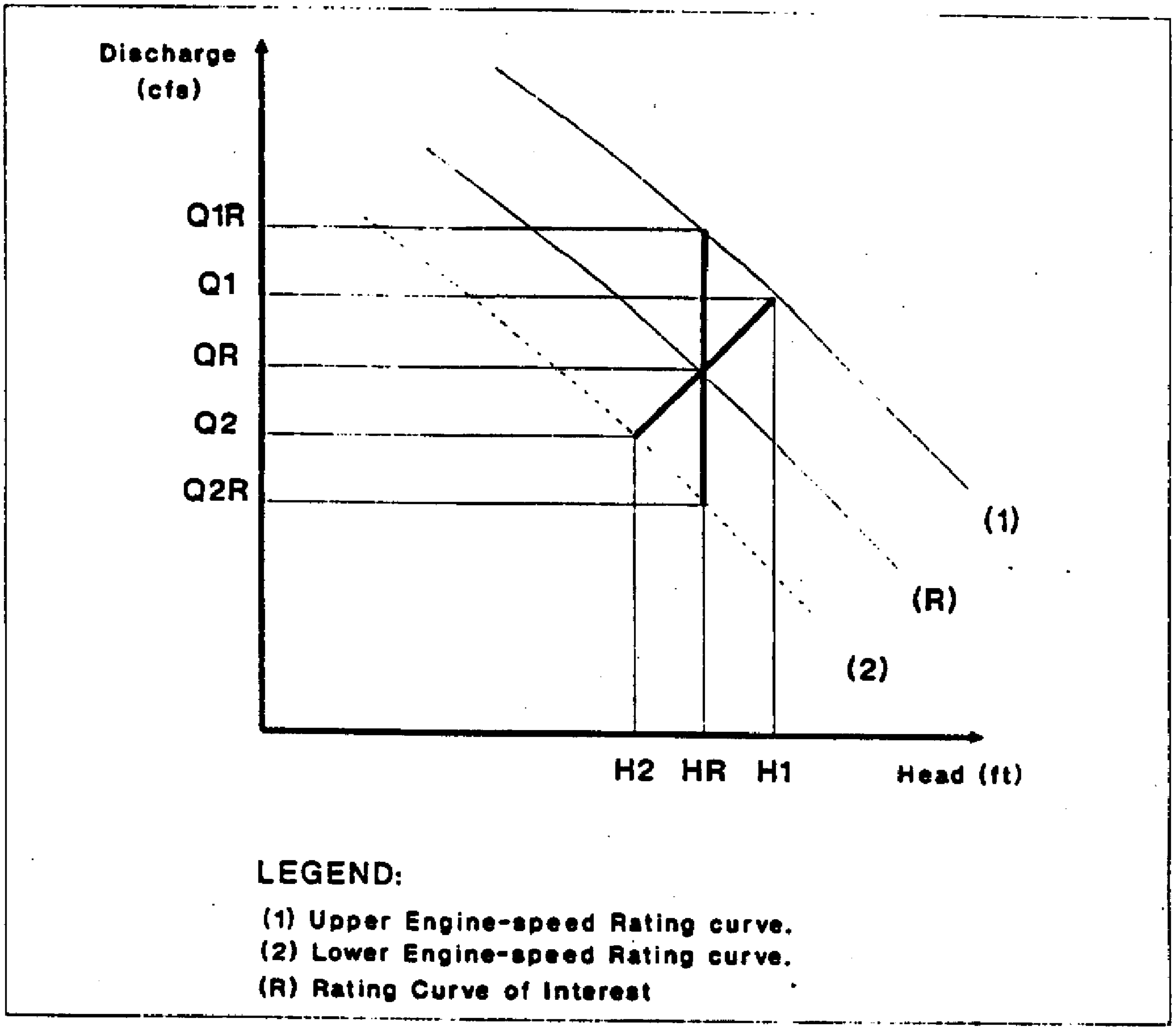

FIGURE 1. Reduction of the Interpolation Range from [(HR,Q1R),(HR,Q2R)] to [(H1,Q1),(H2,Q2)].

The use of H_1 and H_2 in the base rating curves polynomial is appropriate because of the following reasons: (1) from Equation 9, when the engine speeds ω, ω_1, and ω_2 are not much different from one another, the ratios of those speeds are between 0.9 and 1.01, and the heads H_R, H_1, and H_2 are basically the same; therefore, one does not commit an error in using either one of those heads in the flow equation; (2) however, when the speeds are very different from one another, the ratio of the speeds is outside the range defined above; therefore, H_1 and H_2 and not H_R should be used to calculate Q_1 and Q_2.

EXAMPLE OF RATING CURVE CALIBRATION AT PUMP STATION S-8

DESCRIPTION OF PUMP STATION S-8

S-8 is a pump station built on the Miami Canal at the northwest corner of Water Conservation Area 3-A, approximately 15 miles west of U.S. Highway 27 and Pumping Station S-7 within the South Florida Management District (SFWMD, 1990) (see Figure 2). It was designed to remove 3/4-inch of water in 24 hours from the 208-square-mile portion of the Everglades Agricultural Area served by the Miami Canal. S-8 removes excess water from its drainage area and discharges it into Water Conservation Area 3-A.

The operation chart showing the service engine speeds and the corresponding rating curves are shown in Figure 3. The lower-most rating curve was developed for 646 RPM, while the upper-most curve was constructed for 707 RPM. These rating curves are of the rising characteristic type and are third order single variable polynomials in the District FLOW program.

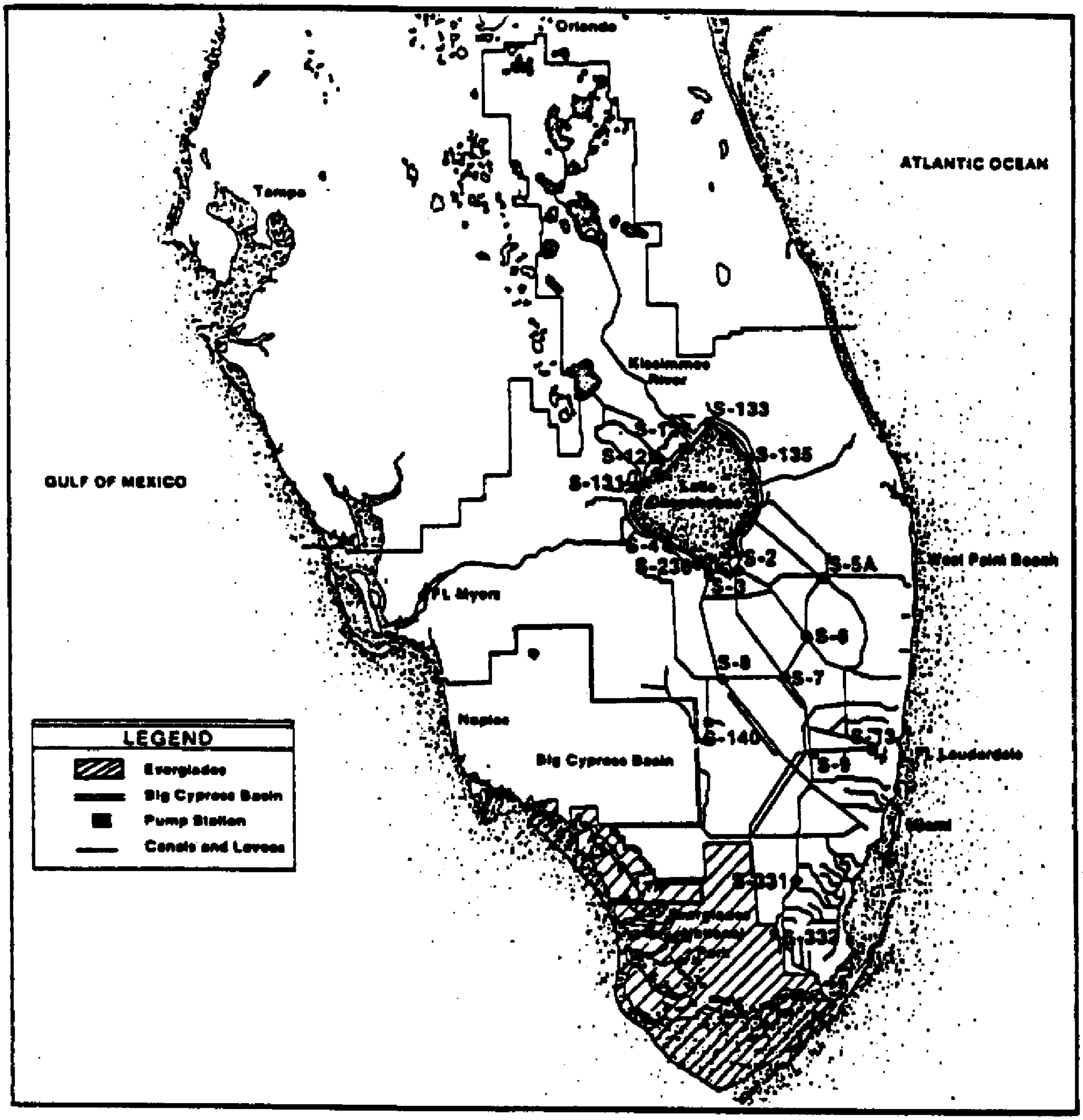

FIGURE 2. Location of Pump Station S-8 within the South Florida Water Management District (Source :. Pump Stations Manual SFWMD).

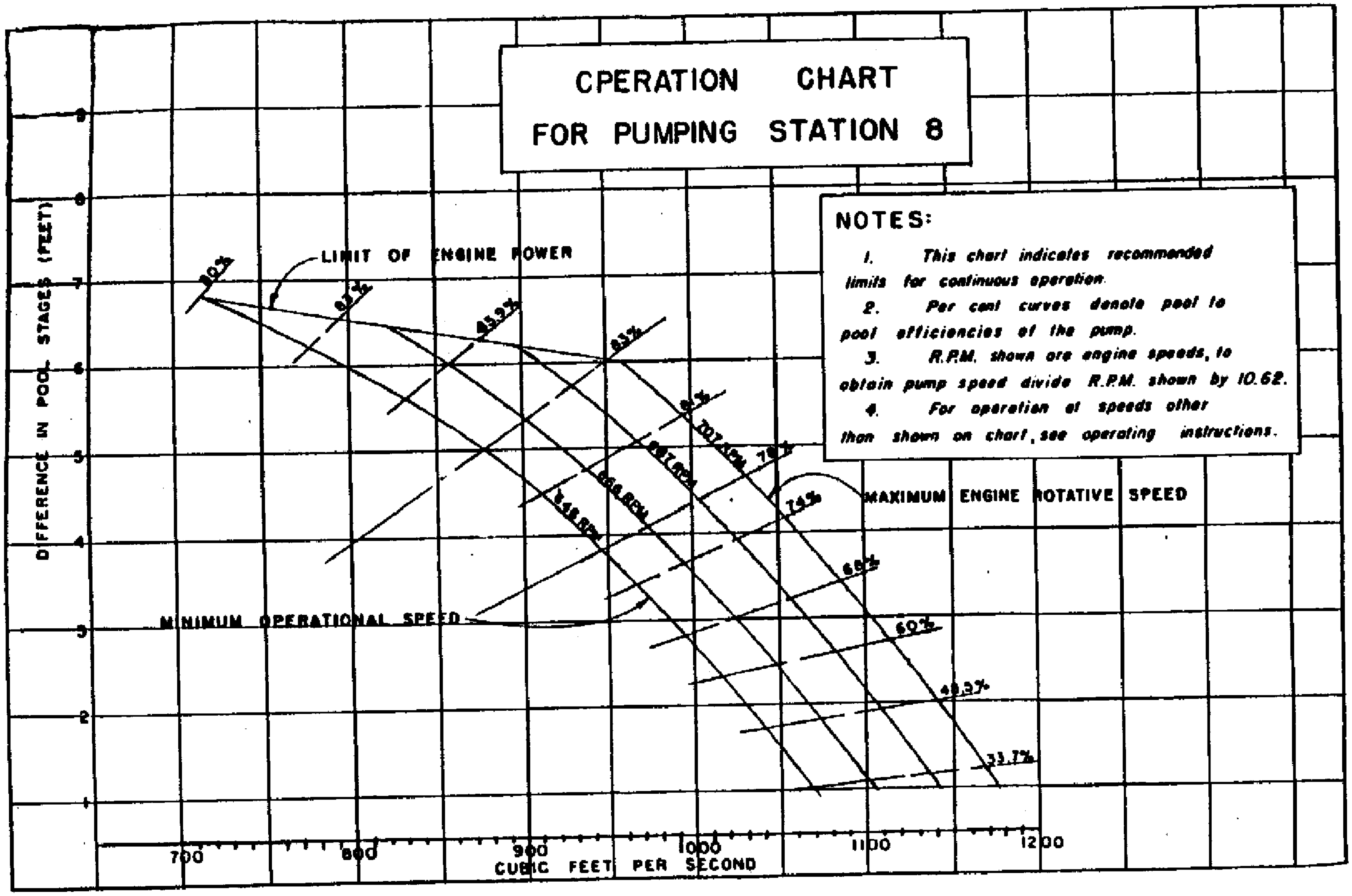

FIGURE 3. Operation Chart for Pumping Station S-8 (Source: US Army Corps of Engineers, Jacksonville, Florida; 1962)

CALIBRATION DATA

Two series of streamgaging data are available for S-8. One series was collected from 1962 to 1968, the other was gathered from 1990 to 1991. Because the two data sets were collected at different periods and potentially at various gaging cross-sections, a preliminary study was conducted to determine if both time series could be combined and used to calibrate the rating curves at S-8. Table 6 and Table 7 contain data used for calibration. The following criterion was developed to investigate the possibility of combining the two different streamgaging data sets for calibration:

1. plot both old and new series on the same graph for visual check of potential outliers,

2. use data from 1990-1991 series to calibrate existing rating curves. Obtain new rating curves. The new series is chosen as the default calibration data set because it describes the most recent trends between measured discharges and discharges calculated using the FLOW program. For most of the pump stations, S-8 included, there is no streamgaging data between the late 1960's and the late 1980's except at S5-A. The old data available, date from the late 1950's to the late 1960's for most pump stations. It does not seem adequate to assume that both series are similar without comparing discharges obtained at the same heads as indicated in the following three steps:

3. use heads of old series in new rating curves to obtain updated calculated discharges.

4. calculate relative errors between calculated discharges of step 2 and corresponding measured discharges of old data set.

5. condition of acceptance of new rating curve as rating curve for both old and new series. If the mean absolute relative error between calculated and measured discharges is less or equal to five percent, and 95% of the data fall within ± 10% of the new rating curve, and 100% of data fall within 15% of the new rating curve,

assume new rating curve is a good prediction of old data set. Therefore, use new rating curve over the combined period of record. For example, if the old data record covered the period 1962-1968 and the new period of record was 1990-1991, then the new rating curve will cover the entire period 1962-1991. If, on the other hand, the conditions above are not met, use the new rating curve for the period it was developed for, and develop a calibrated curve for the old data set, if necessary.

Preliminary Study of the Data

The combined calibration data at 646 RPM for 1962-1968 and 1990-1991, reported in Table 8, were used to plot the graph on Figure 4; while the calibration data at 707 RPM for the same period in Table 9 were used to plot the graph on Figure 5. From these two graphs, the experimental data collected from 1990 to 1991 are shifted downward from the original rating curve as computed with the FLOW program. On the other hand, the data of the period 1962-1968 shifted upward from the original rating curve.

One trend is obvious--the old discharges (1962-1968 data) are generally greater than the new discharges--i.e., the relative error between two discharges for the same head is greater than five percent. During the course of this work, an attempt to explain the discrepancies between the two series of data was made. Factors listed early in the document as being possible causes of errors in rating curve calibration were investigated. It was found that factors such as loss of discharge due to efficiency drop and changes introduced by construction of new structures did not have an effect on calibration. Human induced errors, changes of flow-measuring equipment and changes of flow-measuring cross-sections may have contributed together to give the discrepancies between the new and old data. The idea that possible outliers associated with lower than expected discharges might have been collected at times when heavier than normal debris had occurred at the trash rack will be investigated later. An attempt to combine the two sets of data and use them for analysis was abandoned because the reasons mentioned above. Therefore, only the data collected from 1990 to 1991 was used in this study. It shall be shown later in the document whether that choice was legitimate or not, by application of the criterion of acceptability of old data define earlier this chapter. A relative error analysis on the latter set of data revealed two points $P_1(H=2.40, Q=697)$ and $P_2(H=3.32, Q=841)$ have relative errors equal to 32 percent and 17 percent, respectively. The two points look like potential outliers. However, the verification of the present data collected through streamgaging did not provide enough evidence to automatically reject these points. A sensitivity analysis is conducted later in this report to evaluate the influence of the presence of these points on the overall calibration.

TABLE 6. Measured Streamgaging Data Collected from 1962 to 1968

Measured Speed (RPM)	H_M Measured Head (ft)	Q_M Measured Discharge (cfs)
750.0	0.78	1050.0
707.0	1.65	1165.0
701.0	1.90	1160.0
650.0	2.01	1045.0
650.0	2.47	1030.0
707.0	2.48	1117.0
707.0	2.97	1125.0
650.0	3.50	970.0
600.0	3.91	930.0
600.0	4.17	1015.0
600.0	4.24	1025.0
630.0	4.28	855.0
682.0	4.32	915.0
650.0	4.83	900.0

TABLE 7. Measured Streamgaging Data Collected from 1990 to 1991

Measured Speed (RPM)	H_M Measured Head (ft)	Q_M Measured Discharges (cfs)
700.0	1.25	1218.0
707.0	1.48	1077.0
650.0	1.58	1019.0
707.0	1.65	1068.0
650.0	1.77	976.0
600.0	2.40	697.0
680.0	2.47	1003.0
700.0	2.96	1081.0
700.0	3.16	1022.0
650.0	3.32	841.0
700.0	3.67	1039.0
700.0	4.35	1007.0

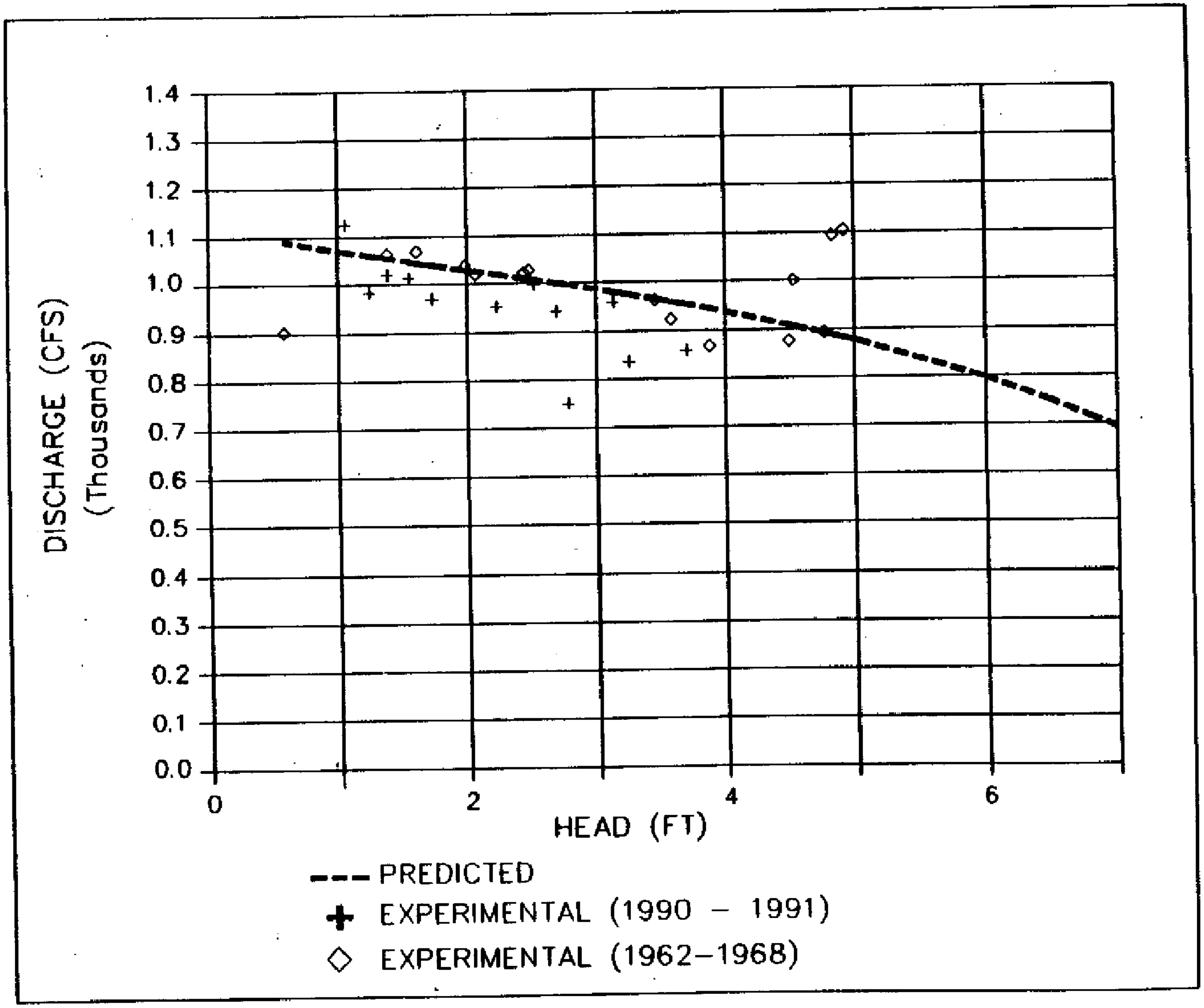

FIGURE 4. **Experimental and Predicted Discharges for Engine-Speed 646 RPM at S-8 for Combined Steramgaging Data of Periods 1962-1968 and 1990-1991.**

TABLE 8. Combined Calibration Data for 1962-1968 and 1990-1991 at 646 RPM.

H Head (ft)	Qx Experimental Discharge (cfs)	Qp Predicted Discharge (cfs)
0.58	904.0	1090.0
1.06	1124.0	1068.0
1.25	984.0	1061.0
1.38	1039.0	1055.0
1.55	1013.0	1047.0
1.61	1020.0	1045.0
1.73	970.0	1040.0
1.99	974.0	1029.0
2.07	960.0	1026.0
2.23	953.0	1020.0
2.44	944.0	1011.0
2.48	944.0	1009.0
2.52	998.0	1008.0
2.69	943.0	1000.0
2.78	750.0	997.0
3.13	959.0	981.0
3.25	836.0	976.0
3.46	906.0	966.0
3.70	858.0	954.0
3.88	874.0	945.0
4.50	922.0	910.0
4.53	986.0	908.0
4.77	956.0	893.0
4.83	1062.0	889.0
4.92	1078.0	883.0

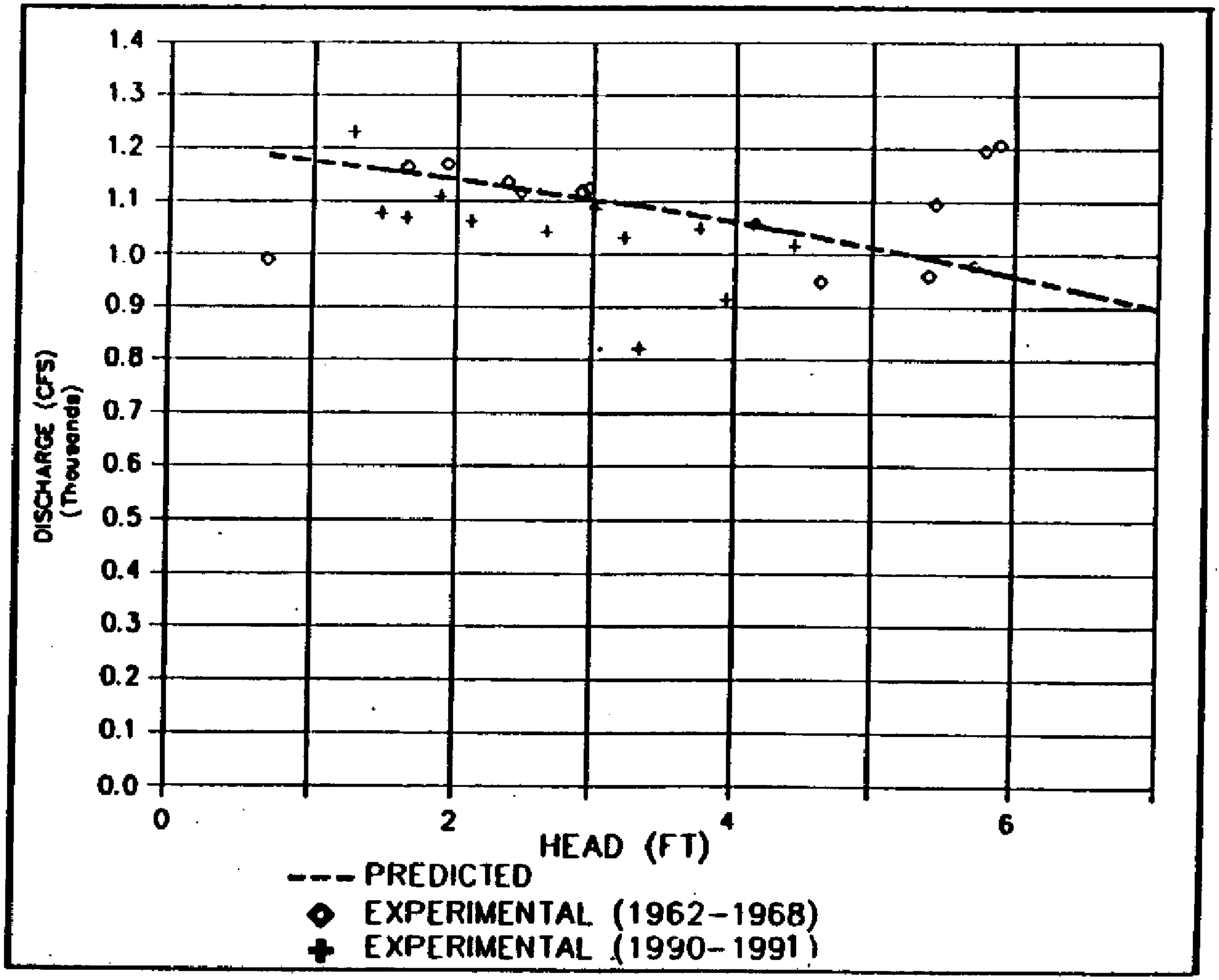

FIGURE 5. Experimental and Predicted Discharges for Engine Speed 707 RPM at S-8 for Combined Data of Periods 1962-1968 and 1990-1991.

TABLE 9. Combined Calibration Data for 1962-1968 and 1990-1991 at 707 RPM.

H Head (ft)	Q_X Experimental Discharge (cfs)	Q_P Predicted Discharge (cfs)
0.69	990.0	1186.0
1.28	1230.0	1166.0
1.48	1077.0	1159.0
1.65	1144.0	1145.0
1.88	1108.0	1145.0
1.93	1142.0	1144.0
2.11	1062.0	1137.0
2.38	1118.0	1127.0
2.48	1106.0	1123.0
2.67	1043.0	1116.0
2.92	1099.0	1107.0
2.97	1101.0	1105.0
3.02	1092.0	1102.0
3.22	1032.0	1095.0
3.33	821.0	1090.0
3.74	1049.0	1073.0
3.94	915.0	1064.0
4.14	1044.0	1055.0
4.44	1017.0	1042.0
4.64	949.0	1032.0
5.39	960.0	994.0
5.43	1096.0	992.0
5.71	979.0	976.0
5.79	1196.0	972.0
5.89	1208.0	966.0

Calibration with the Data Collected from 1990 to 1991

A. Base Rating Curves Calibration

1. The 646 RPM Rating Curve Calibration. The calibration data at 646 RPM in Table 10 is plotted on Figure 6. From Figure 6, the experimental points are shifting downward from the predicted rating curve obtained with the discharge computed with FLOW. Because of that trend, a new rating curve needs to be developed to improve accuracy of computed discharges. The curve fitting methods described previously were applied to the data to obtain a rating curve which satisfies the constraints defined by the criteria of good calibration. The results obtained by those different methods are given below.

1. Results of the Direct Fit Method. A quadratic equation was fitted to the data using the direct least squares method. The following equation:

$$Q_{C646} = 19.77\,H^2 - 169.2\,H + 1216 \qquad (27)$$

in which:

Q_{C646} = calculated discharge at engine speed 646 RPM

H = head

was obtained. Equation 27 could not be accepted because the second derivative condition, which requires that the coefficient of the term H^2 be less than zero, was not satisfied. Figure 7 shows the curve obtained through the direct least squares fit method.

TABLE 10. Calibration Data for 1990-1991 at 646 RPM

Measured Speed (RPM)	H_M Measured Head (ft)	Q_M Measured Discharge (cfs)	H Adjusted Head (ft)	Q_X Experimental Discharge (cfs)	Q_P Predicted Discharge (cfs)
700.0	1.25	1218.0	1.06	1124.0	1068.0
707.0	1.48	1077.0	1.25	984.0	1061.0
707.0	1.65	1068.0	1.38	976.0	1055.0
650.0	1.58	1019.0	1.55	1013.0	1047.0
650.0	1.77	976.0	1.73	970.0	1040.0
680.0	2.47	1003.0	2.23	953.0	1020.0
700.0	2.96	1081.0	2.52	998.0	1008.0
700.0	3.16	1022.0	2.69	943.0	1000.0
600.0	2.40	697.0	2.78	750.0	997.0
700.0	3.67	1039.0	3.13	959.0	981.0
650.0	3.32	841.0	3.28	836.0	976.0
700.0	4.35	1007.0	3.70	858.0	954.0

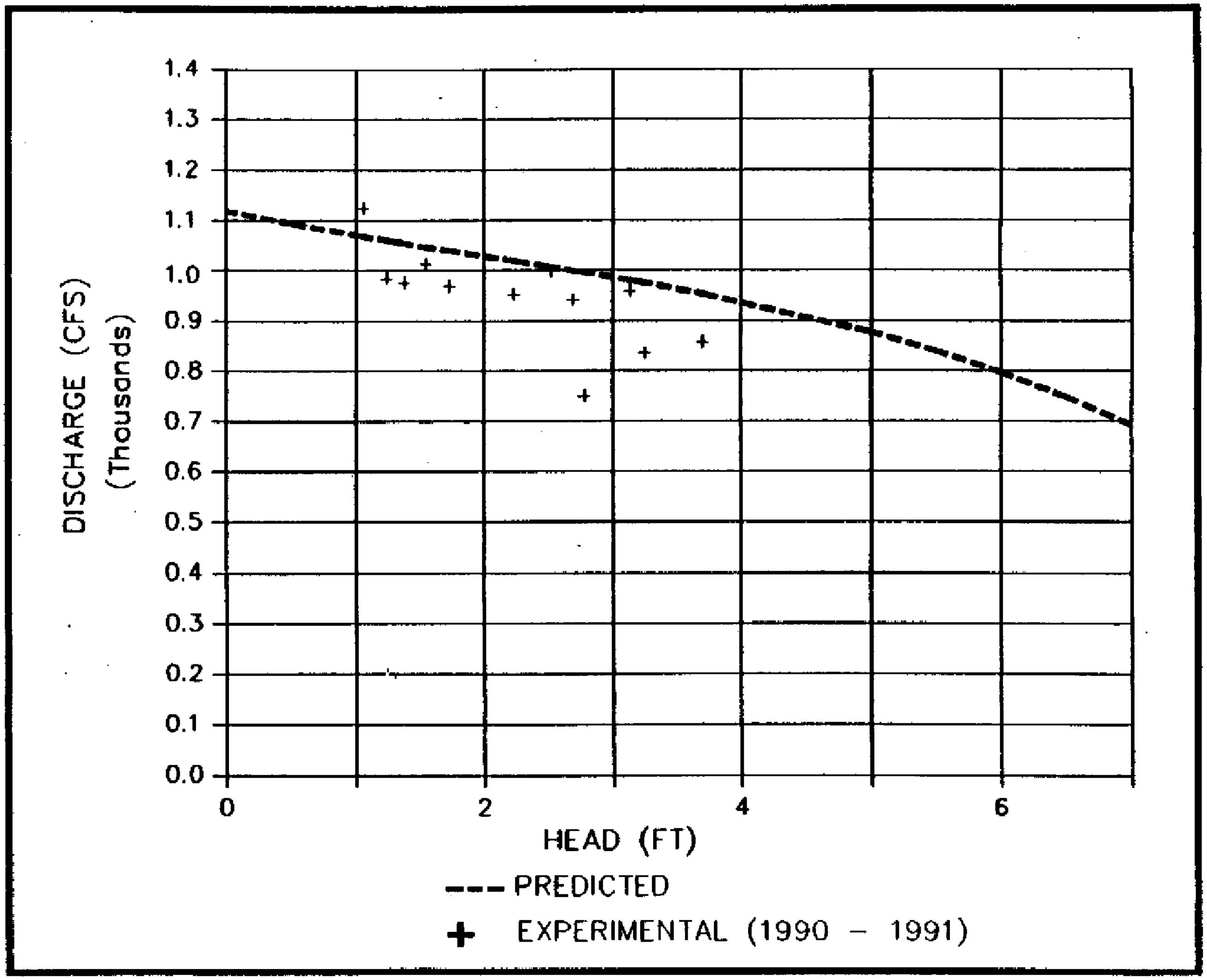

FIGURE 6. Experimental Data and Predicted Rating Curve at 646 RPM at S-8.

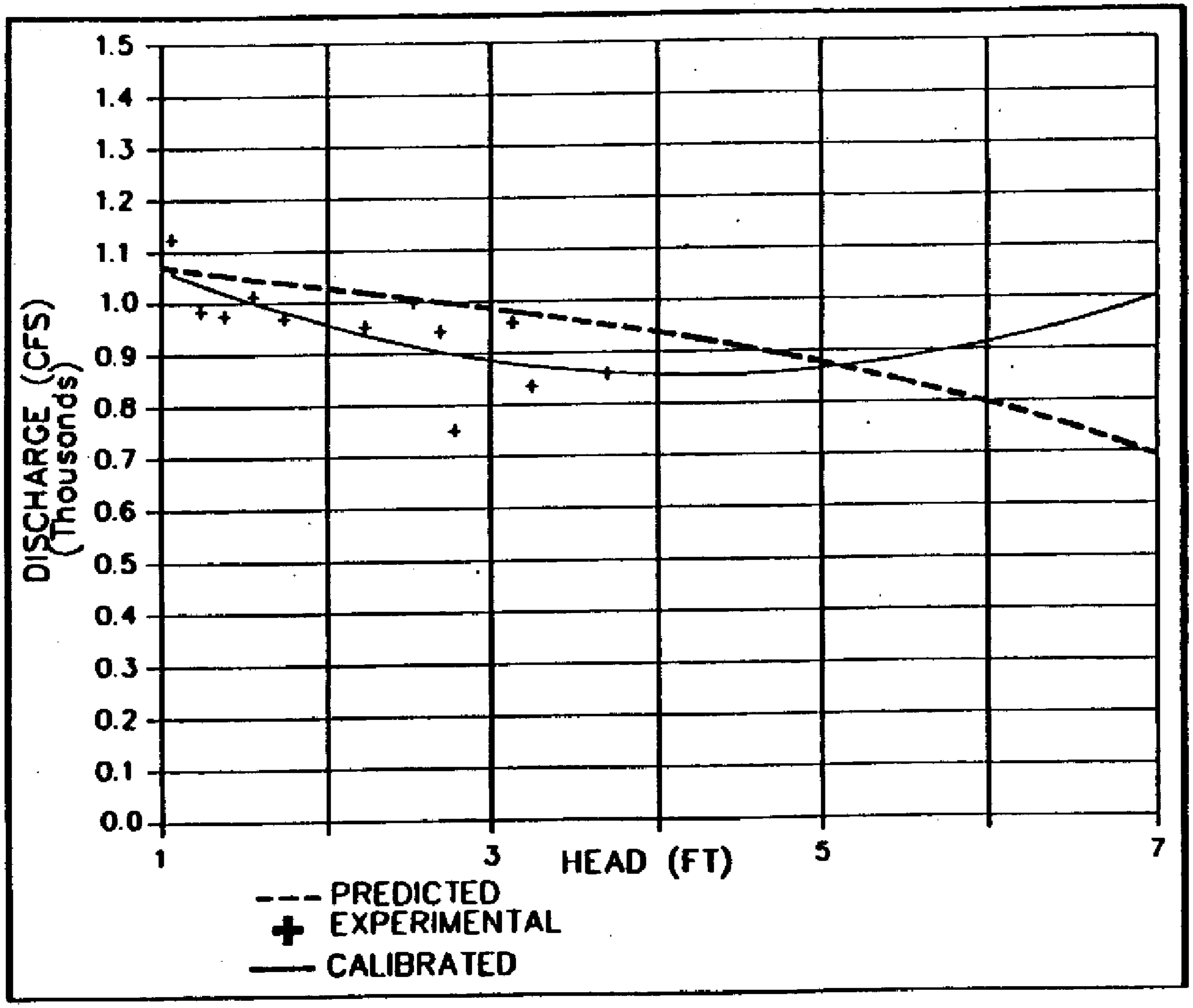

FIGURE 7. Calibrated Rating Curve Obtained by Direct Least Squares Fit Method at Engine Speed 646 RPM at S-8.

2. Results of the "Eyeball Fitted Curve" Method. An eyeball fit curve was drawn to account for the downward shift of the experimental data. In the drawing process, the eyeball curve was forced to asymptomatically match the predicted curve at heads higher than 3.7 feet yet within the range of operation of the pump, i.e., between 0.0 and 7.0 feet. The best eyeball fit curve is shown on Figure 8. The equation of the polynomial is:

$$Q_{C646} = -7.050\,H^2 + 6.766\,H + 1013 \tag{28}$$

in which:

Q_{C646} = calculated discharge rate at engine speed 646 RPM

H = head.

This equation meets the second derivative criterion but does not meet the first derivative condition because the first derivative of the polynomial is zero for H equal to 0.48 foot, a value included within the 0.0 to 7.0 feet range of operation of the pump. Therefore, the polynomial defined by equation 28 cannot be accepted as a new rating curve polynomial.

3. Results of the Reduced Least Squares Method. None of the equations obtained with this method could meet the second derivative criteria.

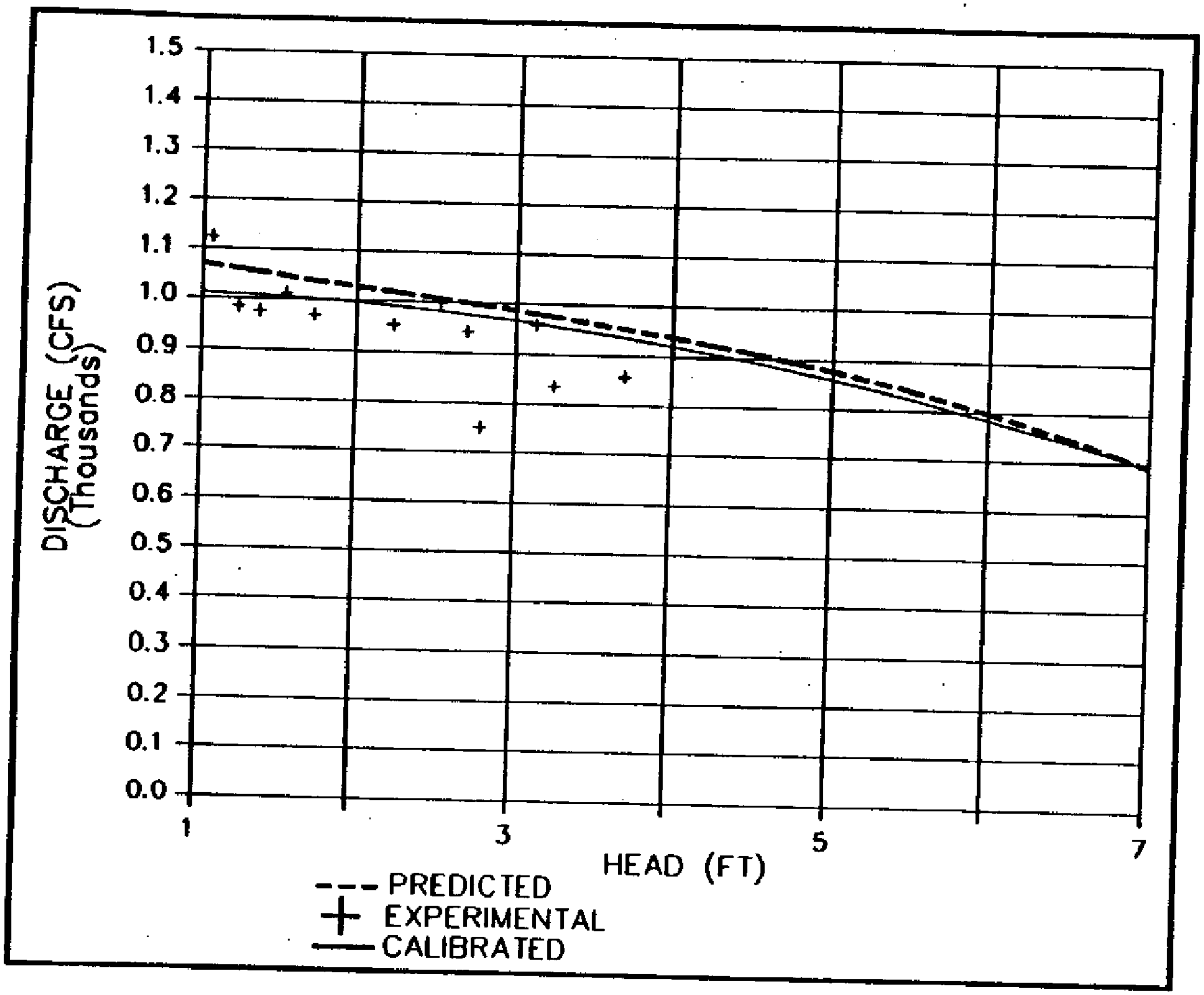

FIGURE 8. Calibrated Eyeball Fit Curve at Engine Speed 646 RPM.

4. The Modified Least Squares Method. A computer program was written to simultaneously solve the three systems of equations previously described. Various matching points were chosen to solve those systems. The following equation:

$$Q_{C646} = -0.602\,H^2 - 63.22\,H + 1100 \qquad (29)$$

in which:

Q_{C646} = calculated discharge at engine speed 646 RPM

H = head

gave the optimum result. That result was obtained with simultaneous equations system #1, with the matching point P(H_0 = 8.0, Q_0 = 556.0). Equation 29 met both the first and the second derivative criteria. The coefficient of correlation, R^2, between calculated and experimental discharge is 0.628. Table 11 compares the relative error, ERR1, between the experimental discharge, Q_X, and the predicted FLOW model discharge, Q_P, and the relative error, ERR2, between the experimental discharge, Q_X, and the calculated discharge, Q_C. Before calibration, only 42 percent of the data met the USGS 5 percent relative error criterion. After calibration, that percentage increased to 55 percent, a 20 percent improvement. Sixty-seven percent (67%) of the data, i.e., eight points out of twelve, showed an error decrease because of calibration. The data of Table 11 is plotted in Figure 9 and shows calibrated match experimental discharges better than the discharges predicted by FLOW.

TABLE 11. Discharge Comparison at 646 RPM for Data Collected from 1990 to 1991 with Coefficient of Determination R^2 = 0.628

H Head (ft)	QX Experimental Discharge (cfs)	Q_P Predicted Discharge (cfs)	Q_C Computed Discharge (cfs)	ERR1 = ABS(Q_X-Q_P)/Q_X	ERR2 = ABS(Q_X-Q_C)/Q_X	ERC = (ERR2-ERR1)/ ERR1
1.06	1124.0	1068.0	1032.0	0.0498	0.0813	63.2
1.25	984.0	1061.0	1021.0	0.0783	0.0369	-52.8
1.38	976.0	1055.0	1012.0	0.0809	0.0369	-54.6
1.55	1013.0	1047.0	1001.0	0.0336	0.0120	-64.2
1.73	970.0	1040.0	989.0	0.0722	0.0197	-72.3
2.23	953.0	1020.0	956.0	0.0703	0.0035	-95.1
2.52	998.0	1008.0	937.0	0.0100	0.0610	508.6
2.69	943.0	1000.0	926.0	0.0604	0.0182	-69.9
2.78	750.0	997.0	920.0	0.3293	0.2265	-31.2
3.13	959.0	981.0	896.0	0.0229	0.0652	184.1
3.28	836.0	976.0	888.0	0.1675	0.0603	-64.0
3.70	858.0	954.0	858.0	0.1119	0.000	100.0

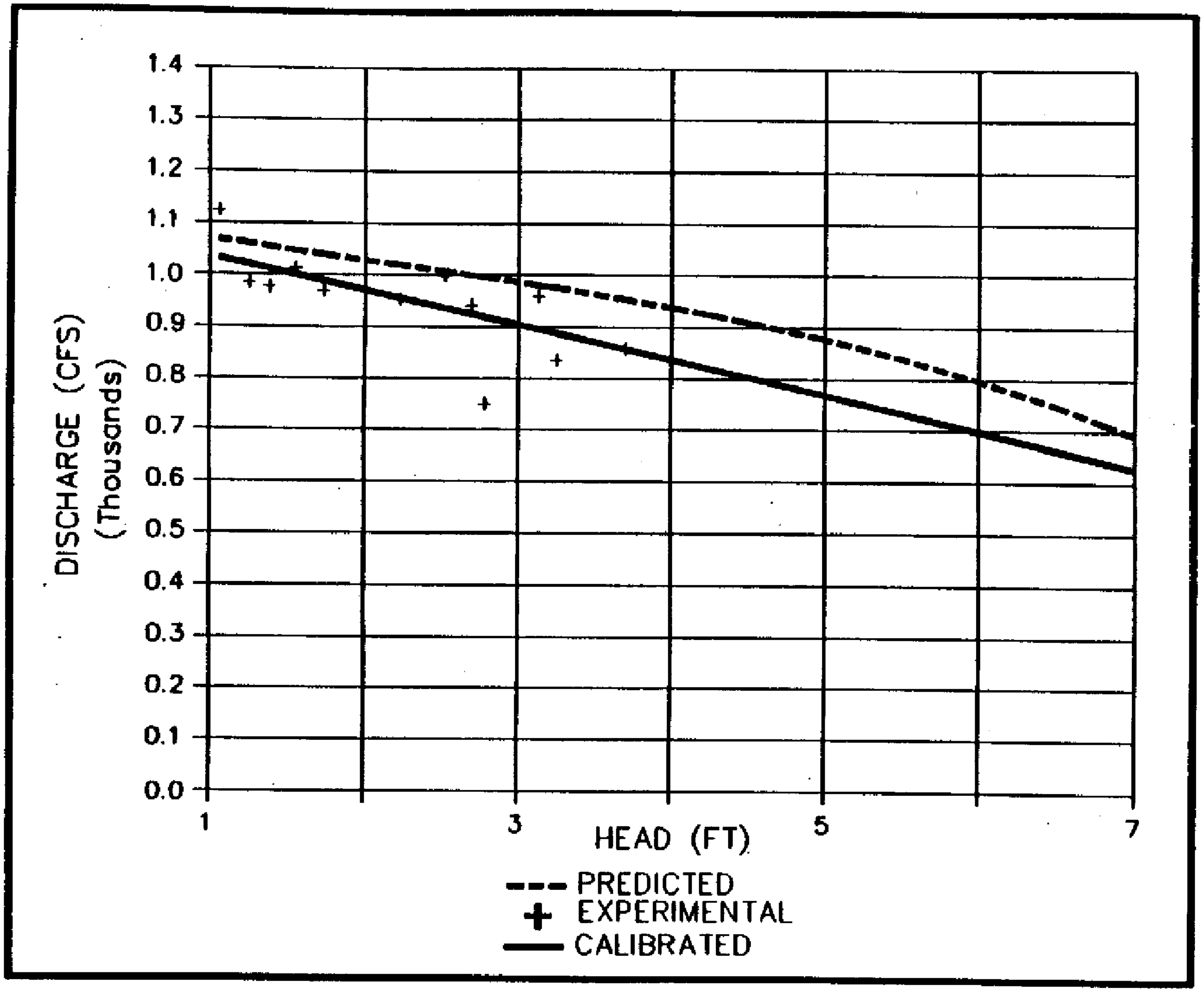

FIGURE 9. The Modified Least Squares Calibrated Rating Curve at 646 RPM.

In summary, because of the improvements noted above and Equation 29 meeting the goodness of fit criteria, Equation 29 can be chosen as the new rating curve.

2. The 707 RPM Rating Curve Calibration The data on Table 12 is plotted in Figure 10. The experimental discharge, like in the 646 RPM rating curve case, shows a downward shift from the original FLOW program rating curve. Calibration of that rating to obtain a better match between measured and estimated discharges was therefore necessary. The results obtained using various curve fitting techniques are reported here.

1. Results of the Direct Fit Method. The quadratic equation fitted to the data is:

$$Q_{C707} = 29.91\,H^2 - 226.2\,H + 1407 \qquad (30)$$

in which

Q_{C707} = calculated discharge at engine speed 707 RPM

H = head.

The concavity of Equation 30 is positive (see Figure 11), and therefore does not meet the second derivative criterion, so Equation 30 cannot be used as a new rating curve equation for the 707 RPM engine speed.

TABLE 12. Calibration Data for 1990-1991 at 707 RPM

Measured Speed (RPM)	HM Measured Head (ft)	QM Measured Discharge (cfs)	H Adjusted Head (ft)	Q_X Experimen-tal Discharge (cfs)	Q_P Predicted Discharge (cfs)
700.0	1.25	1218.0	1.28	1230.0	1166.0
707.0	1.48	1077.0	1.48	1077.0	1159.0
707.0	1.65	1068.0	1.65	1068.0	1154.0
650.0	1.58	1019.0	1.88	1108.0	1145.0
650.0	1.77	976.0	2.11	1062.0	1137.0
680.0	2.47	1003.0	2.67	1043.0	1116.0
700.0	2.96	1081.0	3.02	1092.0	1102.0
700.0	3.16	1022.0	3.22	1032.0	1095.0
600.0	2.40	697.0	3.33	821.0	1090.0
700.0	3.67	1039.0	3.74	1049.0	1073.0
650.0	3.32	841.0	3.93	915.0	1064.0
700.0	4.35	1007.0	4.44	1017.0	1042.0

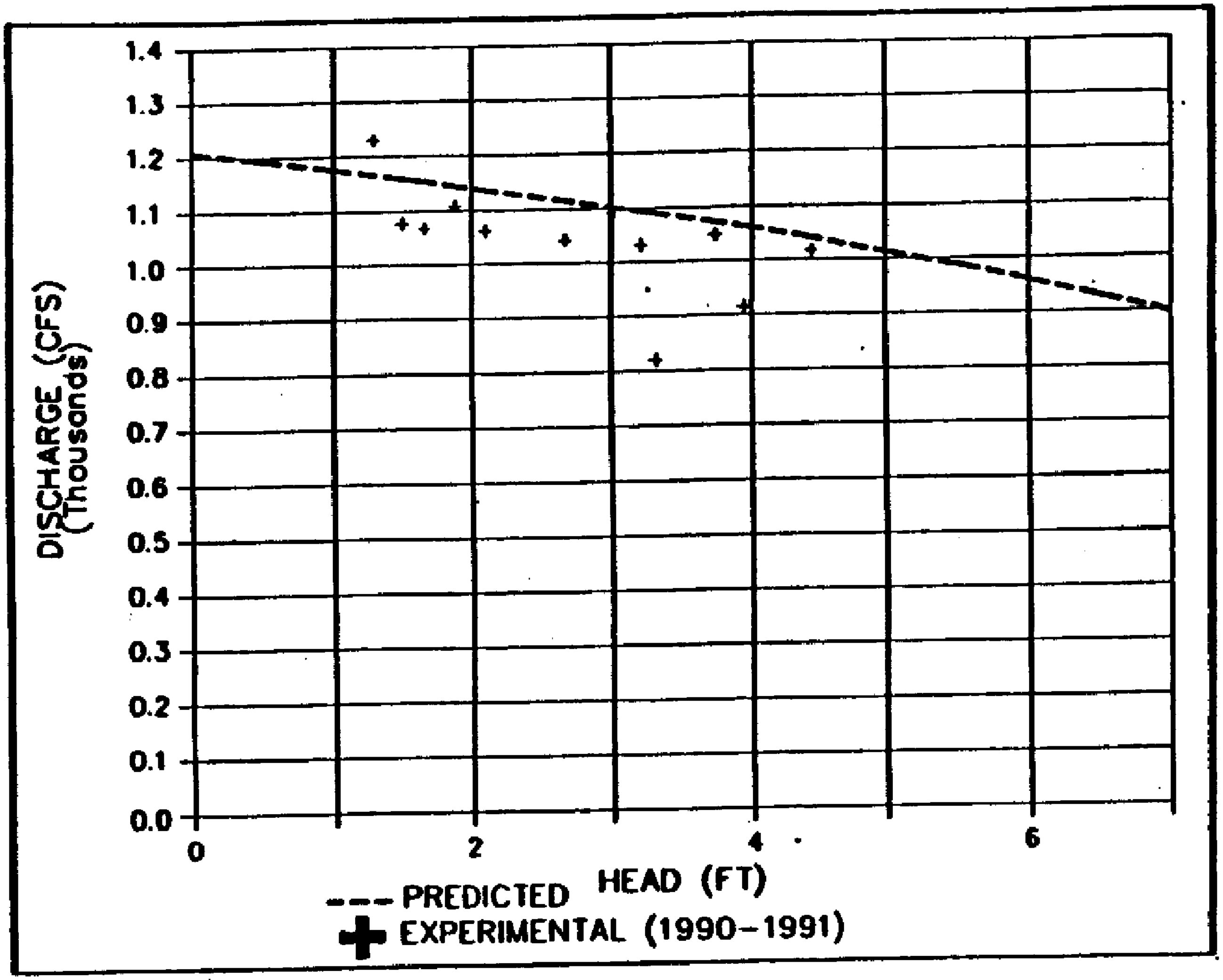

FIGURE 10. Experimental Data and Predicted Rating Curve at Engine Speed 707 RPM at S-8.

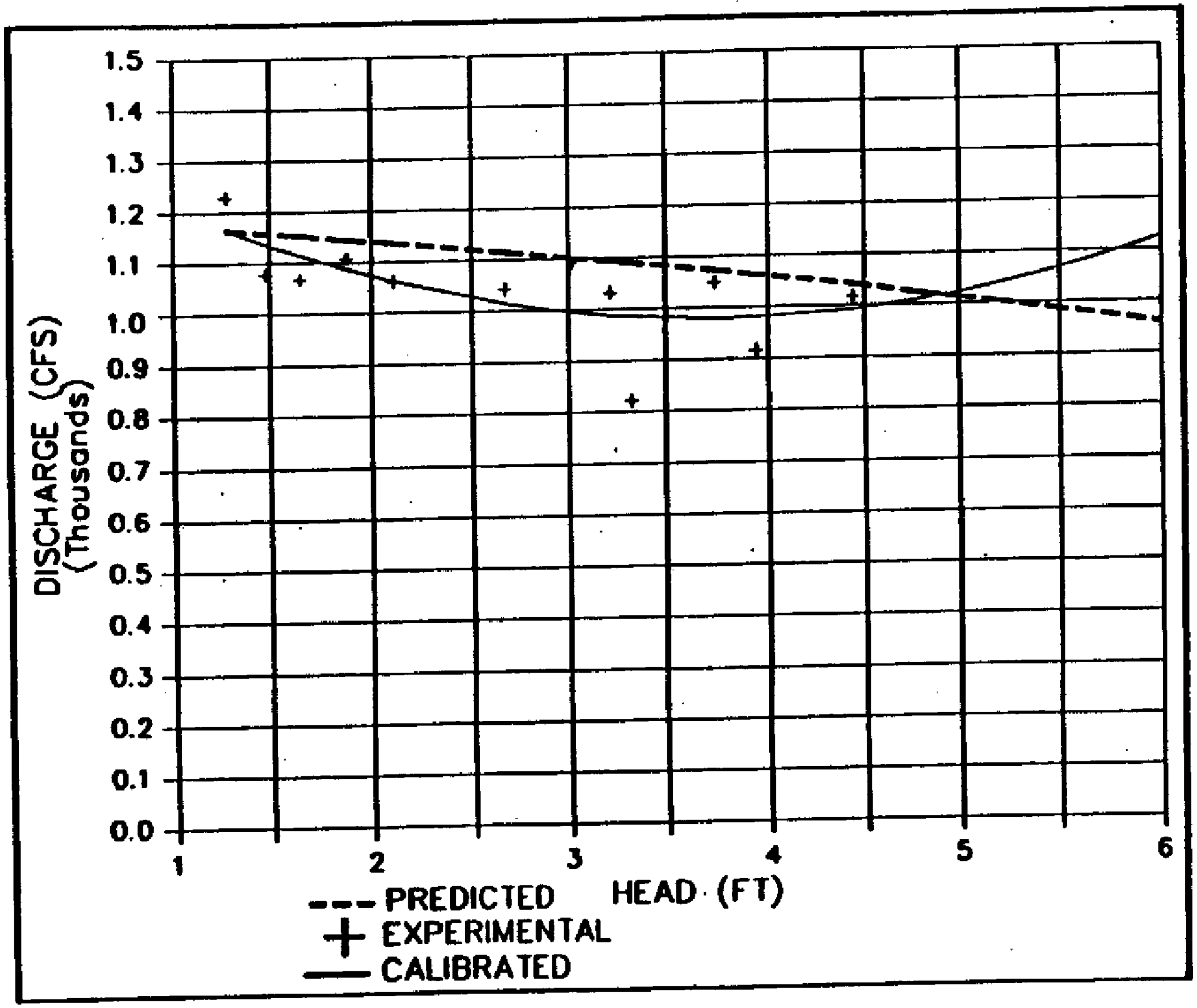

FIGURE 11. Calibrated Rating Curve by the Direct Least Squares Fit at 707 RPM at S-8.

Q_{C707} = calculated discharge at 707 RPM

H = head.

The coefficient of determination R^2 between experimental and computed discharges is equal to 0.629. Equation 32 satisfies the second derivative condition and the first derivative criteria. Table 13 was used to compare the relative errors ERR1 and ERR2 defined as in the case of the 646 RPM rating curve development. Calibration introduced an overall error decrease for 58 percent of the data. The percentage of points meeting the USGS five percent relative error limit increased by 50 percent after calibration. Figure 13 shows a better match between discharges computed using Equation 32 than the predicted discharges obtained with the FLOW model. Equation 32 can be taken as the new rating curve polynomial because it satisfies the goodness-of-fit criteria defined in step 2.

TABLE 13. Discharge Comparison at 707 RPM for Data Collected from 1990 to 1991 with Coefficient of Determination R^2 = 0.629

H Head (ft)	Q_X Experi-mental Discharge (cfs)	Q_P Predicted Discharge (cfs)	Q_C Calibrated Discharge (cfs)	ERR1 = ABS(Q_P-Q_X)/Q_X	ERR2 = ABS(Q_C-Q_X)/Q_X	ERC = (ERR2-ERR1)/ ERR1 %
1.28	1230.0	1166.0	1086.0	0.0520	0.1171	125.2
1.48	1077.0	1159.0	1075.0	0.0761	0.0021	-97.3
1.65	1068.0	1154.0	1065.0	0.0805	0.0252	-96.9
1.88	1108.0	1145.0	1053.0	0.0334	0.0501	50.0
2.11	1062.0	1137.0	1040.0	0.0706	0.0210	-70.3
2.67	1043.0	1116.0	1009.0	0.0700	0.0330	-52.9
3.02	1092.0	1102.0	989.0	0.0092	0.0942	929.9
3.22	1032.0	1095.0	978.0	0.0610	0.0523	-14.3
3.33	821.0	1090.0	972.0	0.3276	0.1838	-43.9
3.74	1049.0	1073.0	949.0	0.0229	0.0953	316.5
3.93	915.0	1064.0	938.0	0.1628	0.0250	-84.6
4.44	1017.0	1042.0	910.0	0.0246	0.1051	327.7

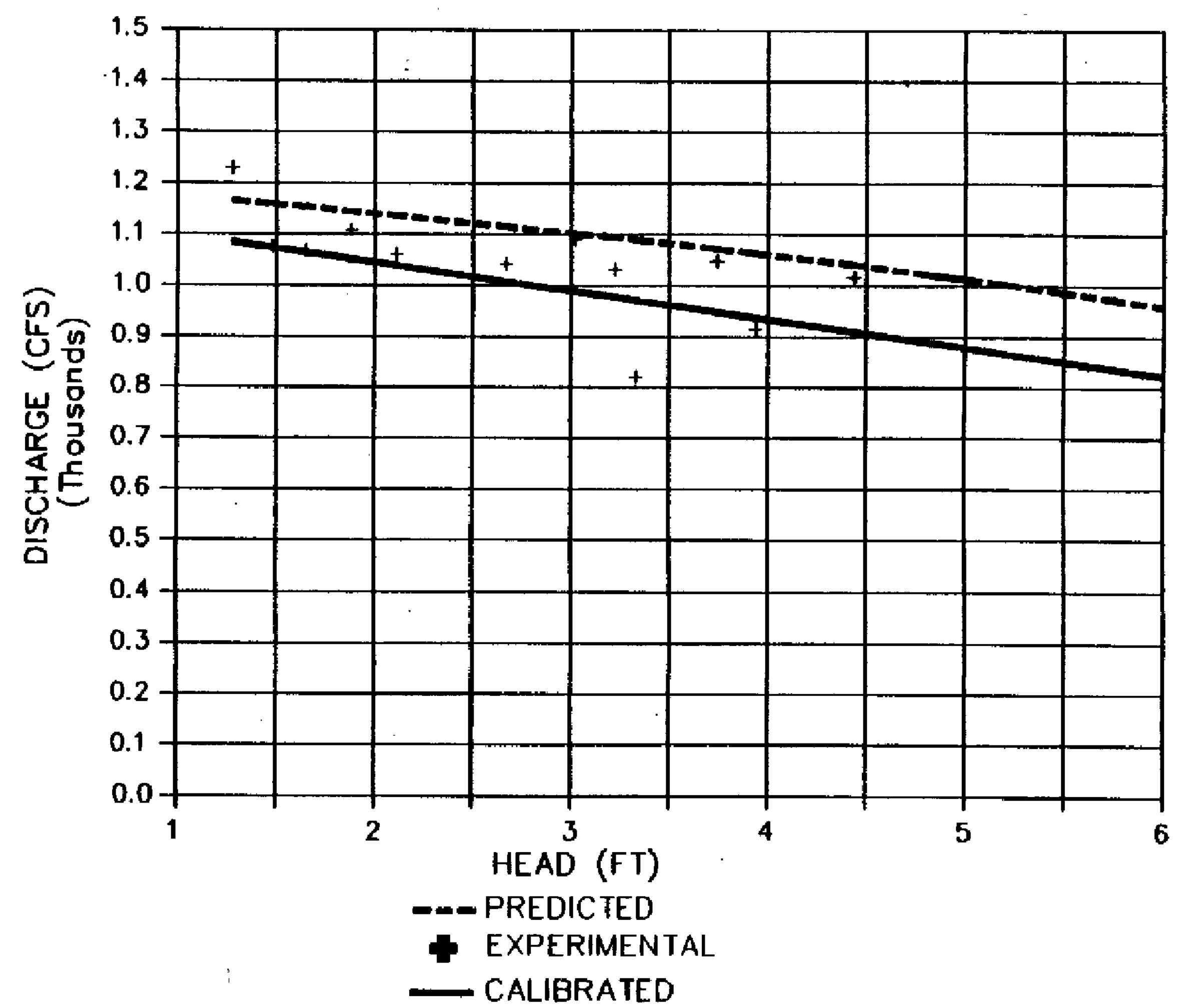

FIGURE 13. The Modified Least Squares Fit Calibrated Rating Curve at 707 RPM at S-8.

B. Verification of the Calibrated Base Rating Curves.

1. Sensitivity Analysis

1.1 Sensitivity Analysis at 646 RPM. Points P_1(H = 2.40,Q = 697) and P_2(H = 3.32,Q = 841) became P_{1646}(H = 2.78,Q = 750) and P_{2646}(H = 3.28,Q = 836) by applying the affinity laws (Beck, 1985) to the heads and discharges defined for points P_1 and P_2. The resulting points P_{1646} and P_{2646} were used for sensitivity analysis in the calibration process. The analysis consisted of evaluating the changes on the R^2 values in a linear regression between the experimental and the calibrated discharges under three calibration conditions: (1) P_{1646} alone was removed from the data before calibration, (2) P_{2646} alone was removed from the data, and (3) both points were simultaneously removed from the data before calibration. Results of the analysis are reported in Table 14. In case (1), a 13 percent improvement in the R^2 value from 0.628 to 0.707 was observed. In case (2), the R^2 decreased 11 percent from its original value. It appears that the removal of R_{646} negatively affects the calibration of the rating curve, therefore, this point needs to be kept. The removal of both points from the data does not affect calibration at all.

1.2. Sensitivity analysis at 707 RPM. By using the affinity laws on the heads and discharges, points P_1(H = 2.40, Q = 697) and P_2(H = 3.32, Q = 841) gave the points P_{1707}(H = 3.33, Q = 821) and P_{2707}(H = 3.93,Q = 915). An analysis similar to the one conducted for speed 646 RPM was performed. The results are reported in Table 15. The removal of P_{1707} caused a 13 percent increase in the R^2 value. Removal of point P_{2707} gave an 11 percent drop of the R^2. When both points are removed simultaneously, the R^2 increased by 1 percent of its original value.

1.3. Conclusion on the Sensitivity Analysis. Removal of the point P_1(H = 2.40, Q = 697) improves the overall R^2 value and gives a better match between calibrated and measured discharges. However, at this phase in the data analysis, this point could not be rejected because the checking of the data, i.e., the verification of the streamgaging notes for that measurement, did not give any evidence that an erroneous value was reported. A specific measurement at 2.40 feet head is needed to confirm or reject the use of that point in future calibration.

TABLE 14. Results of Sensitivity Analysis for Speed 646 RPM at S-8 with R^2 = 0.628

Calibration Conditions	New R^2	Relative Change in R^2
Single point P_1(2.78,750) removed from data	0.707	12.58%
Single point P_2(3.28,836) removed from data	0.559	-10.99%
Both points removed from data	0.628	0.00%

TABLE 15. Results of Sensitivity Analysis for Speed 707 RPM at S-8 with R^2 = 0.629

Calibration Conditions	New R^2	Relative change in R^2
Single point P_1(3.33,821) removed from data	0.709	12.72%
Single point P_2(3.93,915) removed from data	0.561	-10.81%
Both points removed from data	0.637	1.27%

2. Flow Prediction with the Calibrated Rating Curves. The calibrated rating curves at 646 RPM and 707 RPM were used, along with the interpolation scheme described in section two,for calculating discharge in at 680 RPM in order to find out if the calibration of the base curves improved the flow prediction at a given speed. Equation 26 was used for the calculations. For that special case, equation 26 becomes:

$$Q_{680}\ (H_{680}) = 0.5574\,(\,Q_{707}\ \ (H_{707}\,) - Q_{646}\ (H_{646}\,)) + Q_{646}\ \ (H_{646}\,) \qquad (33)$$

in which:

$Q_{680}(H_{680})$ = discharge rate at head H_{680} for engine speed 680 RPM.

H_{680} = adjusted head at 680 RPM.

Q_{707} = discharge computed at head H_{707} with the 707 RPM base rating curve polynomial.

H_{707} = adjusted value of the head H_{680} at the engine speed 707 RPM.

Q_{646} = discharge computed at head H_{646} with the 646 RPM base rating curve.

H_{646} = adjusted value of the head H_{680} at the engine speed 646 RPM.

Table 16 contains parameters used to compute discharges at 680 RPM using calibrated base rating curves. Table 16 shows the results of the flow calculations using the calibrated base curves. Those results showed that, before calibration, 33 percent of the data passed the USGS 5 percent condition compared to 50 percent after calibration. The results, indicated by Table 17 and the better match between calculated and experimental discharges given by the new curves (Figure 14), show that calibration improves discharge predictions at speeds different than those of the base rating curves.

TABLE 16. Key Parameters of Discharge Calculations at 680 RPM using Equation 33

H_{680} Heads at 680 RPM (ft)	H_{707} Adjusted Heads at 707 RPM (ft)	H_{646} Adjusted Heads at 646 RPM (cfs)	Q_{707} Predicted Discharges at 707 RPM (cfs)	Q_{646} Predicted Discharges at 646 RPM (cfs)	Q_{680} Calibrated Discharge at 680 RPM (cfs)
1.18	1.28	1.06	1086.0	1032.0	1062.0
1.37	1.48	1.25	1075.0	1021.0	1051.0
1.53	1.65	1.38	1065.0	1012.0	1041.0
1.73	1.87	1.55	1053.0	1000.0	1030.0
1.94	2.10	1.73	1040.0	987.0	1017.0
2.47	2.67	2.23	1009.0	956.0	985.0
2.79	3.02	2.52	989.0	937.0	966.0
2.98	3.22	2.69	978.0	926.0	955.0
3.08	3.33	2.78	972.0	920.0	949.0
3.46	3.74	3.12	949.0	897.0	926.0
3.63	3.92	3.28	939.0	886.0	916.0
4.10	4.43	3.70	911.0	856.0	887.0

TABLE 17. Discharge Comparison at 680 RPM for Data Collected between 1990 and 1991

H Head (ft)	Q_X Experi-mental Discharge (cfs)	Q_P Predicted Discharge (cfs)	Q_C Calibrated Discharge (cfs)	ERR1 = ABS(Q_P-Q_X)/Q_X	ERR2 = ABS(Q_C-Q_X)/Q_X	ERC = (ERR2-ERR1)/ERR1
1.18	1183.0	1123.0	1062.0	0.0507	0.1021	101.4
1.37	1036.0	1115.0	1051.0	0.0763	0.0144	-81.1
1.53	1027.0	1109.0	1041.0	0.0798	0.0140	-82.4
1.73	1066.0	1102.0	1030.0	0.0338	0.0342	138.5
1.94	1021.0	1094.0	1017.0	0.0715	0.0039	-94.5
2.47	1003.0	1073.0	985.0	0.0698	0.0176	-74.8
2.79	1050.0	1061.0	966.0	0.0105	0.0798	662.1
2.98	663.0	1053.0	955.0	0.5882	0.4400	-25.2
3.08	791.0	1049.0	949.0	0.3262	0.1994	-38.9
3.46	1009.0	1032.0	926.0	0.0228	0.0824	261.40
3.63	880.0	1025.0	916.0	0.1648	0.0405	-75.4
4.10	978.0	1002.0	887.0	0.0245	0.0928	278.3

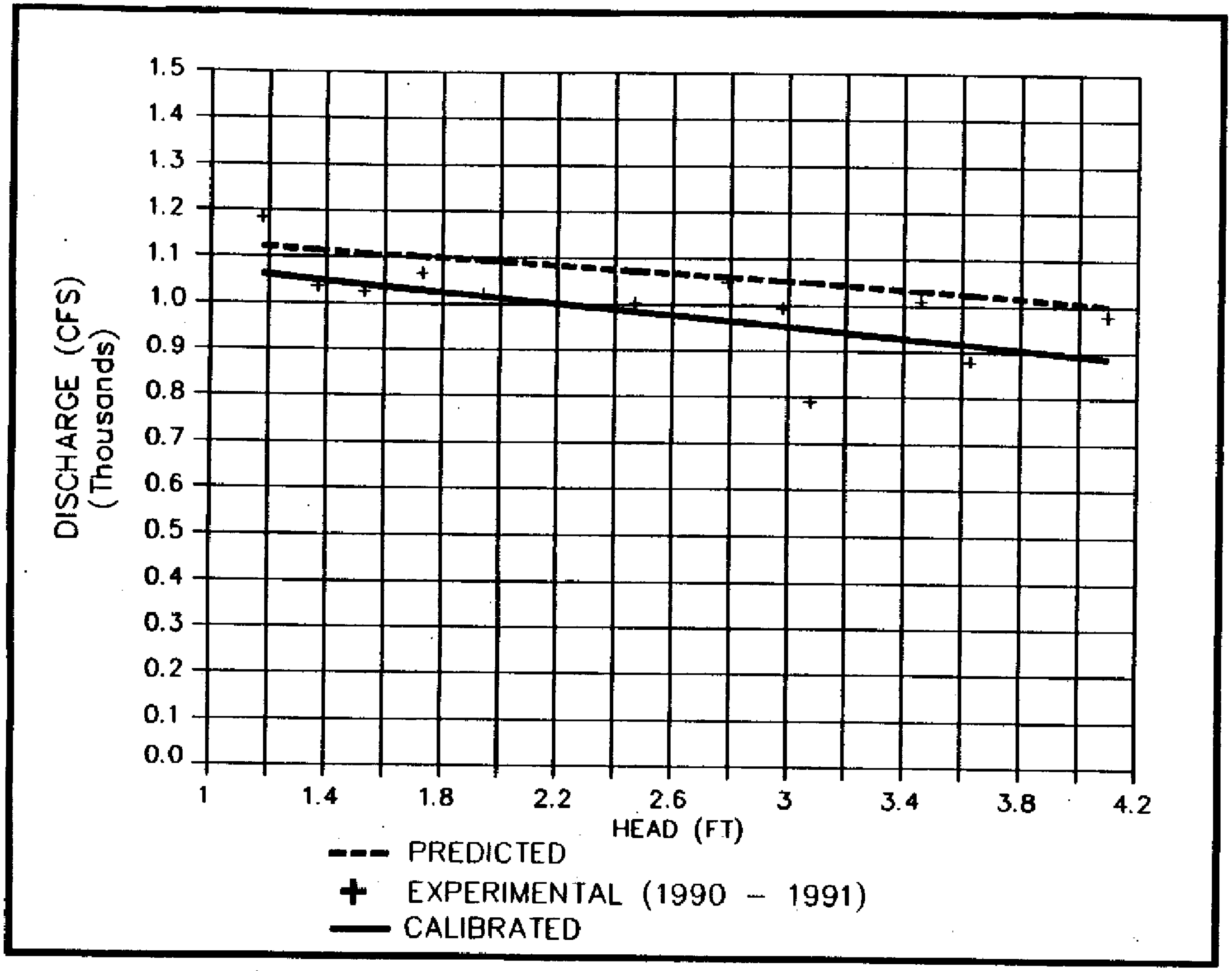

FIGURE 14. Calibrated and Predicted Rating Curves at 680 RPM at S-8

C. Confidence Limits on Rating Curves

1. General Remarks. The SAS software was used to fit a second order polynomial ($Q = aH^2 + bH + c$) to the streamgaging data of S-8. SAS also helped build confidence curves on the regression curve. Two types of confidence intervals were used to obtain those curves. One type of confidence interval (the CLM Confidence Interval) is an interval for the expected value of the mean response. The other type of confidence interval (the CLI Confidence Interval) is an interval for the actual value of the response, i.e., the expected value plus a random error term. The CLI confidence interval is also called the prediction interval for a future observed response.

2. Description of the Confidence Intervals

A) The 95 Percent CLM Confidence Interval on the Mean Response of a Predicted Discharge. The lower limit (L) of the confidence interval is given by:

$$L = Q_{CM} - S\, t_{\alpha/2,n-3} \, (H_0' \, A^{-1} \, H_0)^{0.5} \tag{34}$$

and the upper limit (U) is defined by:

$$U = Q_{CM} + S\, t_{\alpha/2,n-3} \, (H_0' \, A^{-1} \, H_0)^{0.5} \tag{35}$$

in which:

L = lower limit of the confidence interval

U = upper limit of the confidence interval

Q_{CM} = mean response of the predicted discharge for the CLM option.

N or n = number of data in the sample of study

S = standard deviation of discharge values

$t_{\alpha/2,n-3}$ = t value obtained from statistical tables

α = level of confidence (Walpole and Myers, 1978)

H_0' and H_0 are the matrices described by:

$$H_{0'} = \begin{vmatrix} 1 & H & H^2 \end{vmatrix} \quad (36)$$

$$H_0 = \begin{vmatrix} 1 \\ H \\ H^2 \end{vmatrix} \quad (37)$$

A^{-1} = inverse of matrix A defined below

$$A = \begin{vmatrix} N & \sum_{i=1}^{N} H_i & \sum_{i=1}^{N} H_i^2 \\ \sum_{i=1}^{N} H_i & \sum_{i=1}^{N} H_i^2 & \sum_{i=1}^{N} H_i^3 \\ \sum_{i=1}^{N} H_i^2 & \sum_{i=1}^{N} H_i^3 & \sum_{i=1}^{N} H_i^4 \end{vmatrix} \quad (38)$$

H = head

B) The 95 Percent CLI Confidence Interval on a Single Predicted Discharge. The lower limit (L) of the confidence interval is given by:

$$L = Q_{CI} - S t_{\alpha/2, n-3} \quad (1 + H_0' A^{-1} H_0)^{0.5} \tag{39}$$

and the upper limit (U) is defined by:

$$U = Q_{CI} + S t_{\alpha/2, n-3} \quad (1 + H_0' A^{-1} H_0)^{0.5} \tag{40}$$

in which:

Q_{CI} = predicted discharge

the other parameters are the same as previously defined).

3. Results of Data Analysis using SAS. The analysis was performed on the streamgaging data obtained at S-8. The upper and lower confidence curves and the predicted rating curves for speeds 646 and 707 RPM are given on Figures 15, 16, 17, and 18. In this work, it was of interest to know the actual value of the response for each individual predicted value, so the CLI confidence interval option is more convenient. The majority of points are within the confidence curves. More discharge data will be collected to verify the acceptability of the points outside the confidence curves.

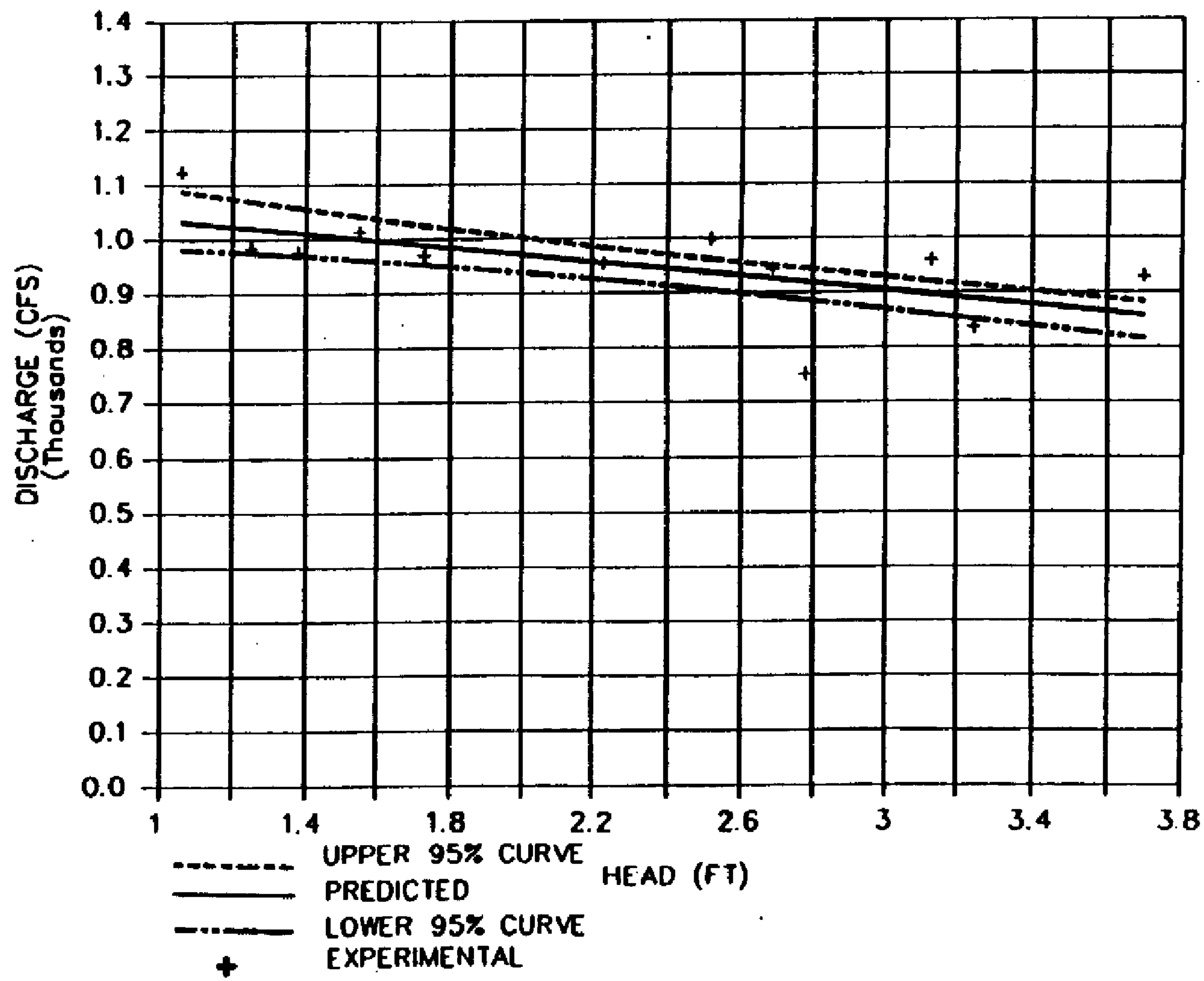

FIGURE 15. Lower and Upper (CLM) 95% Confidence Curves, Predicted Rating Curve and Experimental Data for 646 RPM at S-8

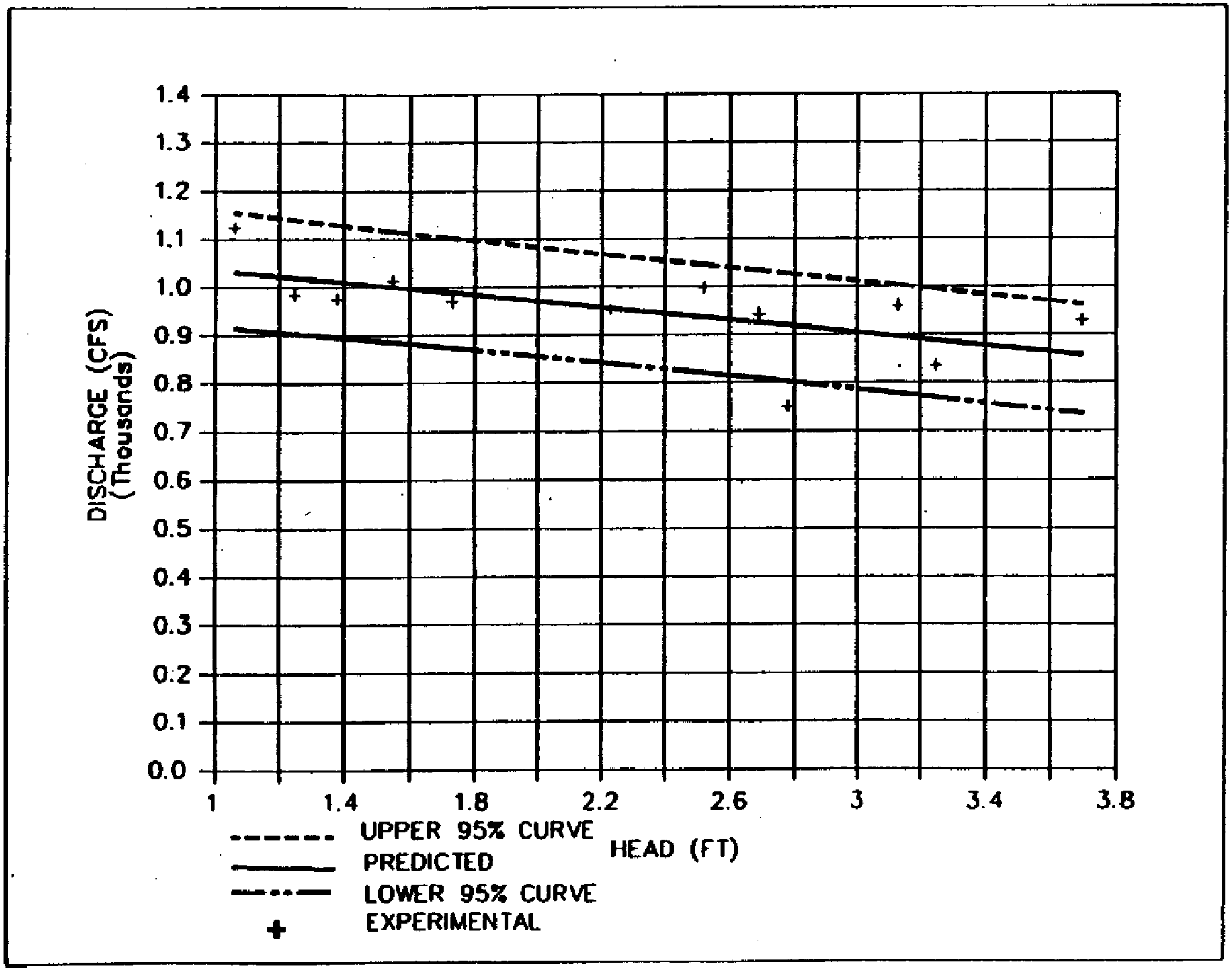

FIGURE 16. Lower and Upper (CLI) 95% Confidence Curves, Predicted Rating Curves and Experimental Data for 646 RPM at S-8.

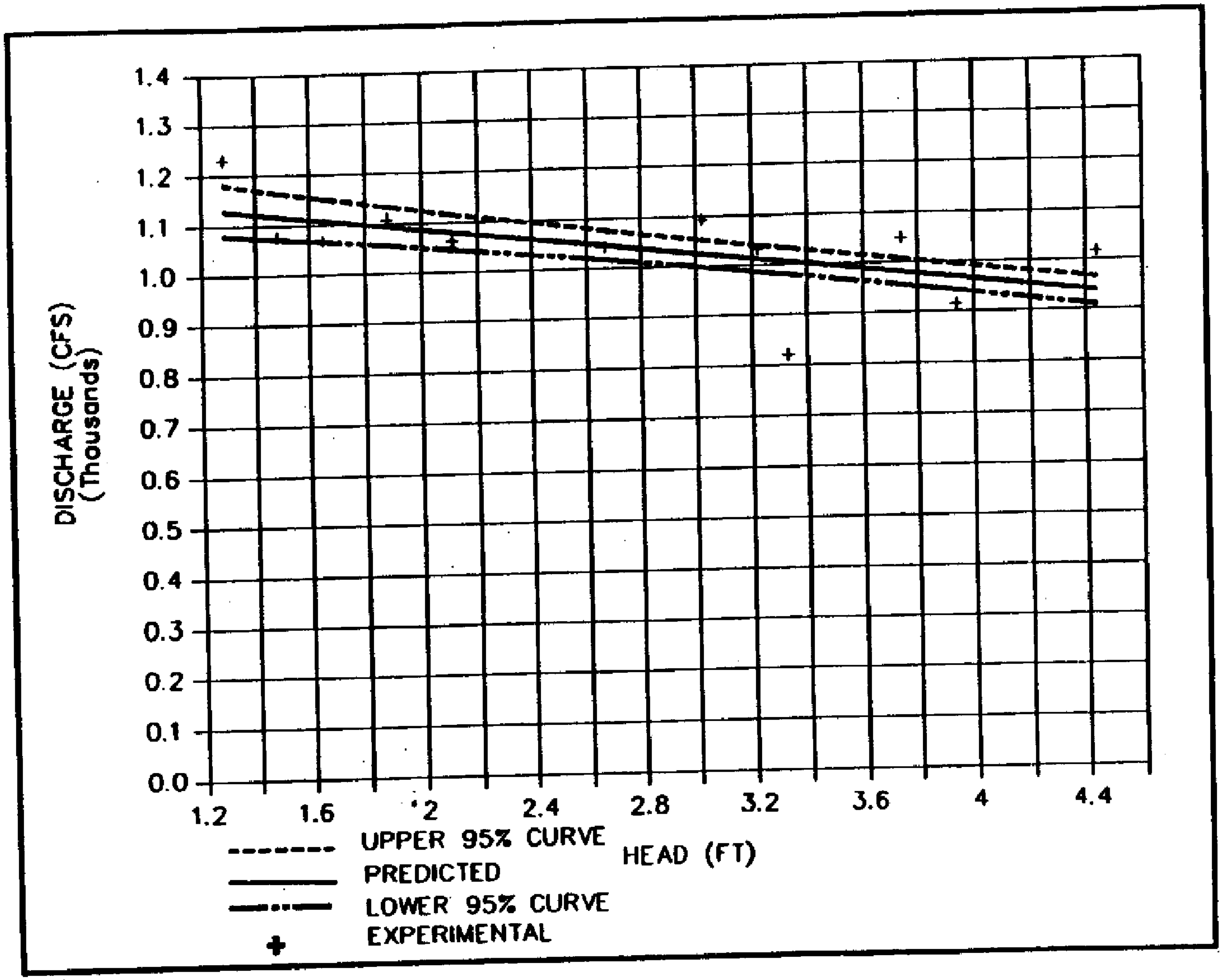

FIGURE 17. Lower and Upper (CLM) 95% Confidence Intervals, Predicted Rating Curve and Experimental Data for 707 RPM at S-8.

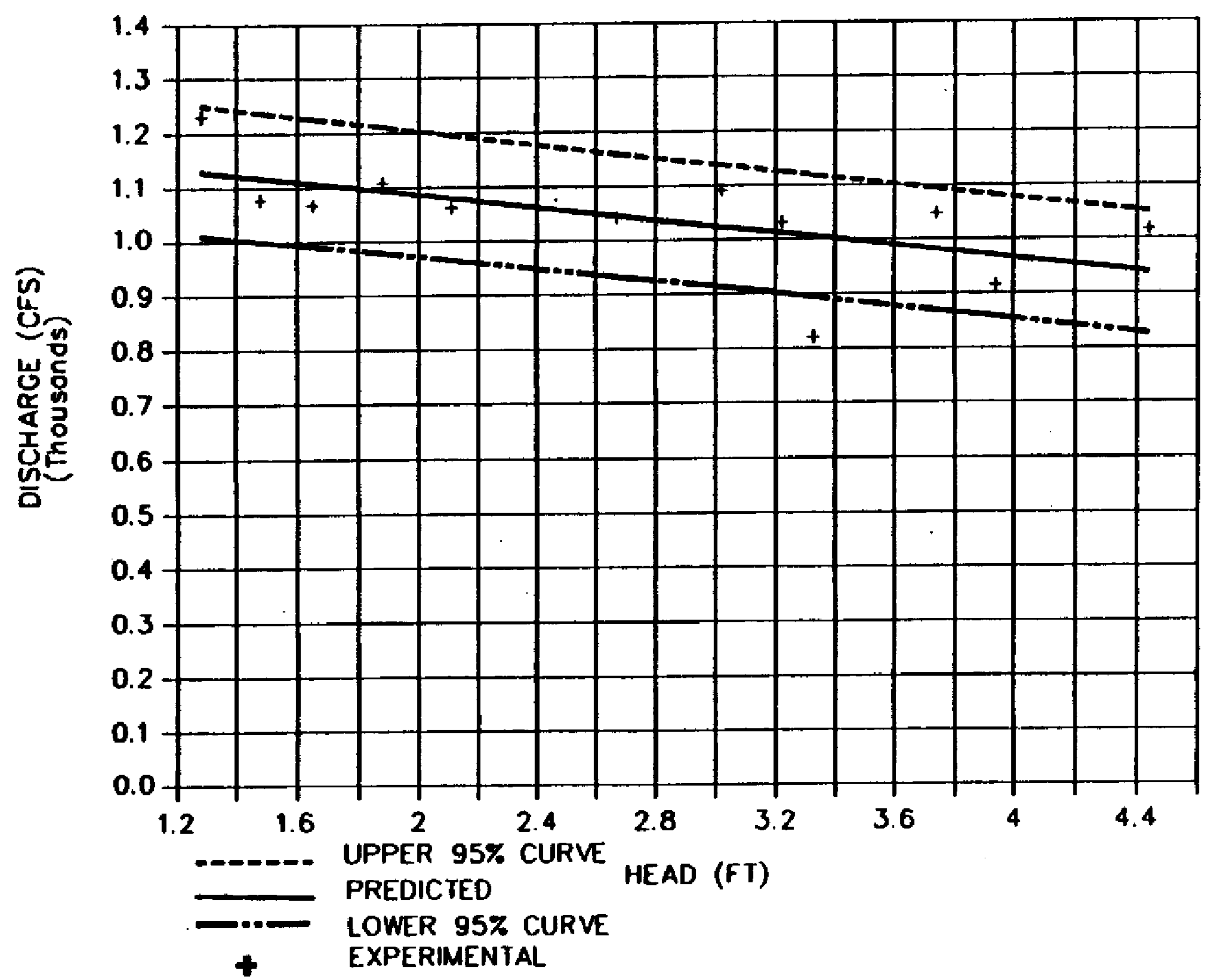

FIGURE 18. Lower and Upper (CLI) 95% Confidence Intervals, Predicted Rating Curve and Experimental Data for 707 RPM at S-8.

D. Estimation of Differences Between Old and New Data.

After the new rating curves were obtained, they were used to explain the causes of discrepancies between the data collected from 1962 to 1968 and those gathered from 1990 to1991. The first topic investigated was the effect of trash accumulation on head losses and the second topic was the verification of whether it was legitimate to combine old and new data for calibration or not. The main problem encountered in evaluating the first topic was the absence of time series of debris-induced headwater drops. However, after gathering information from the Operations Division staff, it was found that periodical estimations of headwater drop due to debris were made. Those estimates ranged from a quarter of a foot (three inches) to a third of a foot (four inches). Headwater drops of about two feet were verbally reported to have occurred at S-9, but not at S-8. Based upon these findings, an analysis was conducted to study the effects of headwater drop on rating curve calibration. The hypothesis stated earlier in this document concerning the decrease of discharge due to higher head losses was investigated, for assumed values of headwater drop.

Three pool-to-pool head differences DH = 1.25, 2.78 and 3.28 ft (measured data for 646 RPM engine speed) were chosen. Assumed values of headwater drops were also given (row 1, Tables 18, 19 and 20). New heads associated with those headwater drops were calculated (row 2, same tables). Those heads were used in FLOW to calculate the new discharge Q_N (row 3, same tables). Row 4, Tables 18, 19 and 20, contains the relative errors between each new discharge Q_N and the measured discharge for a given original head (Q_M is constant, Q_N varies with each assumed headwater drop). This relative error describes the relative variation of the measured discharge if headwater drop was accounted for. The other relative error (row 4) described the departure from the discharge Q_P, calculated with the original head, when no headwater drop was accounted for.

The results of this study are reported in Tables 18, 19, and 20. The tables show that the relative error between Q_M and Q_N decreases as the headwater drop increases. A value is reached where the relative error is equal to zero. This value indicates how much of headwater drop is required for the measured and calculated discharges to be the same. This value is equal to 1.70 ft for DH = 1.25 ft, 3.72 ft for DH = 2.78 ft, and 2.26 ft for DH = 3.28 ft. These relative errors indicate that accounting for headwater drop gives a better match between measured and calculated discharges. However, the values of headwater drop giving a good match between calculated and measured discharge are too large compared to the value of 0.25 to 0.3 ft usually observed at S-8. Unless further analysis proves otherwise, the headwater drop does not seem to explain by itself the departure between the calculated and measured discharges at S-8.

The second hypothesis was investigated as follows: an updated discharge was obtained by introducing a head having a corresponding discharge value in the old data set (1962 - 1968) into the equation of the base rating curve polynomial obtained with the 1990-1992 data). The updated discharge (Q_U) thus obtained represents a discharge strongly influenced by the new trends in the data. The comparison between the old discharge and the new discharge at the same head gives an idea of how much the discharges have changed between those two data collection periods for the 646 and 707 RPM engine speeds rating curves. Tables 21 and 22 show the relative error differences between updated and measured discharges. The mean absolute relative error between these two discharges was higher than five percent; therefore, the 1962-1968 and the 1990-1991 data should not be combined for calibration (see good calibration criterion developed in early chapters of this document). The new rating curve should therefore be used for the

1990-1991 data and the old FLOW program rating curve should be used for the 1962-1968 data.

From this analysis, it appears that the decision of not combining both series of data to develop the rating curves was justified.

TABLE 18. Effect of Debris Related Headwater Drop on Discharge Calculation at S-8 for DH = 1.25 ft .

DH = 1.25 ft: Pool-to-Pool Head Difference Q_M = 984 cfs :Measured Discharge for DH = 1.25 ft Q_P = 1061 cfs:Computed Discharge for DH = 1.25 ft						
Headwater Drop (ft)	0.01	0.10	0.25	0.5	1	1.70
New Head After Drop (ft)	1.26	1.35	1.50	1.75	2.25	2.95
Q_N : Discharge Computed With New Heads (cfs)	1060	1056	1050	1039	1019	984
Relative Error (%): $(Q_N-Q_M)/Q_M$	7.72	7.32	6.71	5.59	3.56	0.00
Relative Error (%): $(Q_N-Q_P)/Q_P$	-0.09	-0.47	-1.04	-2.07	-3.96	-7.26

TABLE 19. Effect of Debris Related Headwater Drop on Discharge Calculation at S-8 for DH = 2.78 ft

DH = 2.78 ft: Pool-to-Pool Head Difference Q_M = 750 cfs :Measured Discharge for DH = 2.78 ft Q_P = 997 cfs:Computed Discharge for DH = 2.78 ft						
Headwater Drop (ft)	0.01	0.10	0.25	1.00	2	3.72
New Head After Drop (ft)	2.79	2.88	3.03	3.78	4.78	6.50
Q_N : Discharge Computed With New Heads (cfs)	996	992	986	950	893	750
Relative Error (%): $(Q_N-Q_M)/Q_M$	32.80	32.27	31.47	26.67	19.07	0.00
Relative Error (%): $(Q_N-Q_P)/Q_P$	-0.10	-0.50	-1.10	-4.71	-10.43	-24.77

TABLE 20. Effect of Debris Related Headwater Drop on Discharge Calculation at S-8 for DH = 3.28 ft.

DH = 3.28 ft: Pool-to-Pool Head Difference Q_M = 836 cfs :Measured Discharge for DH = 3.28 ft Q_P = 976 cfs:Computed Discharge for DH = 3.28 ft						
Headwater Drop (ft)	0.01	0.10	0.25	0.5	1	2.26
New Head After Drop (ft)	3.29	3.38	3.53	3.78	4.28	5.54
Q_N : Discharge Computed With New Heads (cfs)	975	970	963	950	923	836
Relative Error (%): $(Q_N-Q_M)/Q_M$	16.63	16.03	15.19	13.64	10.41	0.00
Relative Error (%): $(Q_N-Q_P)/Q_P$	-0.10	-0.61	-1.33	-2.66	-5.43	-14.34

TABLE 21. Heads, Experimental, and Updated Discharges, and Relative Errors for Data Period 1962-1968 for Engine Speed 646 RPM.

H Heads (ft)	Q_X Experimental Discharge (cfs)	Q_U Updated Discharge (cfs)	Relative Error $(Q_U-Q_X)/Q_U$
0.58	904.0	1063.0	-0.1760
1.38	1064.0	1012.0	-0.0492
1.61	1069.0	997.0	-0.0677
1.99	1039.0	972.0	-0.0647
2.07	1021.0	967.0	-0.0533
2.44	1024.0	942.0	-0.0799
3.46	964.0	874.0	-0.0933
3.58	923.0	866.0	-0.0618
3.88	867.0	846.0	-0.0246
4.50	877.0	803.0	-0.0840
4.53	1001.0	801.0	-0.1995
4.77	894.0	785.0	-0.1222
4.83	1093.0	781.0	-0.2858
4.92	1104.0	774.0	-0.2986
N/A	N/A	Mean Absolute Relative Error	0.1186

TABLE 22. Heads, Experimental and Updated Discharges, and Relative Errors for Data of the Period 1962-1969 for Engine Speed 707 RPM.

H Heads (ft)	Q_X Experiment al Discharge (cfs)	Q_U Updated Discharge (cfs)	Relative Error $(Q_U-Q_X)/Q_X$
0.69	990.0	1165.0	-0.1767
1.65	1165.0	1108.0	-0.0492
1.93	1170.0	1091.0	-0.0677
2.38	1137.0	1064.0	-0.0644
2.48	1117.0	1058.0	-0.0530
2.92	1120.0	1031.0	-0.0792
2.97	1125.0	1028.0	-0.0860
4.14	1055.0	957.0	-0.0925
4.64	949.0	927.0	-0.0232
5.39	960.0	881.0	-0.0820
5.43	1096.0	879.0	-0.1982
5.71	979.0	862.0	-0.1199
5.79	1196.0	857.0	-0.2837
5.89	1208.0	851.0	-0.2959
N/A	N/A	Mean Absolute Relative Error	0.1194

CONCLUSIONS

Applying pump affinity laws to adjust the measured heads and discharges at various speeds into their corresponding values at a given speed of interest was fundamental to this study. This procedure allowed the plotting of the data at a given engine speed to check the match between experimental and calculated discharges and to spot shifts and various trends on the rating curves. This procedure also helped develop an improved interpolation scheme between the two base rating curves by shortening the interpolation range between curves.

An example of calibration studies of the rating curves was carried out at pump station S-8 using the procedure described above. A direct fit of a polynomial to the data for the base rating curves at speeds 646 RPM and 707 RPM failed to give new rating curves, showing a good match between experimental and calculated discharges. An eyeball-fitted curve method also failed to give results meeting the goodness-of-fit criteria defined in the study. A second-order polynomial was fitted to the data by solving a system of three equations. Two of these three equations are obtained from the least squares method system of equations; the third equation was obtained by forcing the calibrated curve to match the predicted FLOW program curve at a point where no experimental discharge was available. The two new second-order polynomials met the goodness of fit criteria set in the study. Those curves were also used to predict discharges at 680 RPM for verification. The results showed that before calibration only 33 percent of the data points met the USGS 5 percent relative criteria; that percentage increased to 50 percent after calibration of the base curve. Although this increase is not exceptional, the addition of future data points will greatly improve the rating curves because the new data will not only help confirm or negate the use of data showing unusual departure from the rating curve, but define the rating curve for heads of the pump station operation with no discharge data.

RECOMMENDATIONS

The first recommendation is to collect data at heads for which no measured discharge data are available, and at heads showing a strong trend, i.e., pronounced upward or downward shifts from the existing rating curves. At S-8, discharge data should therefore be collected at 2.40 feet head at 646 RPM because a sensitivity analysis revealed the removal of the discharge at that head improved the R^2 value between computed and measured discharges. Data are also needed at heads lower than 1.0 foot or higher than 3.7 feet when the engine speed is 646 RPM, and at heads lower than 1.28 feet or higher than 4.0 feet when the engine speed is 707 RPM. Another recommendation is to continue streamgaging to eliminate random errors in the data as much as possible, and to extend the calibration method developed in this study to other pump stations.

Although there are no more than 20 data points for the calibration process, the obvious downward shift of the discharges from the FLOW model rating curve seems to indicate that a modification of the base rating curve at S-8 is necessary.

GLOSSARY

Confidence Interval. Interval defining a domain of accuracy or confidence on a data set.

Efficiency Coefficient. Coefficient rating the performance of a pump. The efficiency coefficient is usually giving in percentage. An efficiency coefficient between 90 percent and 100 percent means that the pumps are performing at optimum capacity.

Experimental Discharge. Discharge obtained by adjusting the measured streamgaging discharge by using pump affinity laws.

FLOW Program. Computer program used by the Data Management Division to compute flow data.

Predicted Discharge. Discharge computed using the Data Management Division FLOW program.

Rating Curve. Plot of stages or pool-to-pool head differences between headwater and tailwater versus discharge.

Streamgaging. The standard streamgaging flow measurement method consists of choosing a stream cross-section downstream or upstream of the structure of interest and measuring the discharges at that cross-section. The measurement procedure consists of dividing the cross-section into a number of subsections along the width of the canal, determining the average velocity at each subsection, and summing the partial products of the subsectional areas and the average velocities. The instrument generally used to obtain the velocity is the Price Meter. That meter has a wheel which turn proportionally to the velocity of water. A calibrated relation between water velocity, the number of rotations of the wheel submerged at a desired depth and the time to obtain the number of rotations of the wheel gives the average velocity at the desired cross-section. Generally velocities taken at 0.2 D and 0.8 D (D = measured subsectional depth) are averaged to obtain the subsectional velocity. The reference mentioned in this document gives more information on the streamgaging techniques.

t-Statistics. A statistical method used to define the distribution of data in a given data set.

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LIST OF SYMBOLS

a = constant in a second order polynomiaL.
A = matrix used to determine confidence limit curves on a given rating curve.
A^{-1} = inverse of matrix A.
a_0 = Coefficient in power function $Q = a_0H^r$
b = coefficient in a second-order polynomial.
c = coefficient of a second-order polynomial.
$c_0,...,c_9$ = regression coefficients.
E_0 = calibration efficiency coefficient.
E = reference calibration efficiency coefficient.
ERC = ratio (ERR2-ERR1)/ERR1.
ERR1 = relative error between experimental and predicted discharges.
ERR2 = relative error between predicted and calibrated discharges.
H = head difference between headwater stage and tailwater stage;
H_{fact} = head factor;
H_i = head for measurement i;
H_M = measured head;
H_{I0} = head matrix used in the computation of confidence limit curves on a given rating curve;
H_{I0}' = inverse of matrix H_{I0};
H_0 = head at matching point between calibrated and predicted rating curves;
H_{00} = null hypothesis in statistical hypothesis analysis;
H_{11} = alternative hypothesis in statistical hypothesis analysis;
H_1 = measured head at engine speed ω_1;
H_2 = adjusted head at engine speed ω_2;
H_{646} = adjusted head value of head H_{680} at the engine speed 646 RPM;
H_{680} = adjusted head at 680 RPM;
H_{707} = adjusted head value of head H_{680} at the engine speed 707 RPM;
L = lower limit of confidence interval;
N = number of data points in sample of study;
P_i = power input to pump engine system;
Q = discharge rate;
Q_C = computed or calibrated discharge;
Q_{CI} = computed discharge for SAS CLI option;
Q_{CM} = computed discharge for SAS CLM option;
Q_{C646} = calibrated or computed discharge at engine speed 646 RPM;
Q_{C707} = calibrated or computed discharge at engine speed 707 RPM;
Q_i = discharge rate for measurement i;
Q_M = measured head;
Q_X = experimental discharge (measured discharge after adjustment by using affinity laws);
Q_{XBAR} = average of all experimental discharge;
Q_P = predicted discharge (discharge calculated with FLOW);

= discharge at matching point between calibrated and predicted rating curves;

Q_1 = discharge calculated with the lower limit rating curve;

Q_2 = discharge calculated with the upper limit rating curve;

Q_{646} = discharge calculated at head H_{646} with the 646 RPM base rating curve;

Q_{680} = discharge calculated at head H_{680} for engine speed 680 RPM;

Q_{707} = discharge calculated at head H_{707} with the 707 RPM base rating curve;

Q_{1R} = discharge for the upper engine speed for the reference head H_R;

Q_{2R} = discharge for the lower engine speed for the reference head H_R;

Q_U = updated discharge for pump efficiency calculation;

r = exponent of power function;

R^2 = coefficient of determination in a regression analysis;

S = standard deviation for sample of study;

SSE = sum of squared errors in the least squares analysis;

t = value of t statistics obtained from statistical tables;

T = t-Statistics parameter for hypothesis analysis;

U = upper limit of confidence interval;

X = head parameter ($X = H/H_{fact}$);

X_{avg} = mean of sample of N data points;

Y = pump speed parameter;

α = level of confidence in statistical analysis;

η = pump efficiency coefficient;

η^1 = pump efficiency coefficient at 646 RPM;

η^2 = pump efficiency coefficient at 707 RPM;

ω = pump engine speed;

ω^1 = pump engine speed at 646 RPM;

ω^2 = pump engine speed at 707 RPM;

μ = true population mean of discharge data;

μ^o = reference mean (usually 0);