TECHNICAL MEMORANDUM

DATA ANALYSIS TO DETECT RAINFALL CHANGES IN SOUTH FLORIDA

by

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CONCLUSIONS

With the assumption of data independence and time series stationarity, changes in rainfall characteristics were quantified. The conclusions are:

1. The maximum difference in the records for pre- and post-change point occurred around 1970.
2. Annual rainfall was about 5 inches per year less in the period after 1970, compared to the period prior to 1970. This reduction came from drier and shorter wet seasons, less heavy storms, and/or less tropical cyclone rainfall.
3. There was a significant change in the variation coefficient of daily maximum rainfall annual series that may affect the storm frequency analysis.
4. The Kissimmee River Valley and the southwest corner of the District showed the most significant changes.

FURTHER STUDIES

Treat the rainfall data as a non-stationary series to find out if:

1. any trend exists,
2. any change in trend has occurred, or
3. if forecast models for planning purposes can be developed.

I. PURPOSE

The recent climatological stresses on the District system have forceably brought attention to the importance of climate to the well being of society. It has long been suspected that climate was subject to systematic variation, but in the absence of a well developed body of theory, it has been more expedient to treat climatological quantities as random variables. Perhaps it is now time to attempt to build such systematic characteristics as can be defined, however roughly, into our hydrologic system considerations. The purpose of this work is to detect if there were any significant rainfall pattern changes in this century. Based on historical rainfall record analysis, statistical significance of change, if any, is qualified.

II. DATA SOURCE

Daily and monthly rainfall data available in the South Florida Water Management District Rainfall Data Base are the data used in this analysis.

III. DETECTING THE TIME OF CHANGE IN THE MEAN OF ANNUAL RAINFALL

A. Data Treatment

1. Monthly rainfall data from stations with over 50 years of record are used for this study.
2. The monthly rainfall is summed up to yearly total for analysis.
3. When missing data occur:
   (a) The year with missing data was deleted, but the position of the missing data-year was kept in the time series.
   (b) The missing data were filled in with mean rainfall of the month.

   Both of these methods were done to compare the results.

B. Method of Analysis

The method of analysis is based on lecture notes from the computer workshop in "Statistical Hydrology," Colorado State University (1).

1. Assumptions:

   (a) The input data, annual rainfall in this case, is independent.

   (b) Rainfall from each station is independent; hence, data from each station is treated as a single series.

2. Question to be answered:

   Given annual rainfall of Xj. j=1,...n. what is the most likely time (T) that a change in the mean of Xj occurs between the two series, Xj. j=1,...T and Xj. j=T+1,....n? The time of change is detected by using Bayesian analysis of posterior distribution of time of change. A computer program for the analysis is attached in Appendix A.

C. Results

Thirty-eight (38) rainfall stations have monthly rainfall records of more than 50 years. Most of them have missing monthly rainfall. When missing data years were deleted, but the positions were kept in the analysis, 31 stations showed detectable changes of mean at some point of time; and 7 stations did not show a significantly detectable time at which change of mean might occur. The time distribution of probable year of change is shown in Figure 1. The analysis is done at 90% confidence level for the Bayesian interval estimate.

When the missing data was filled with the mean value, the time of change was much less detectable.

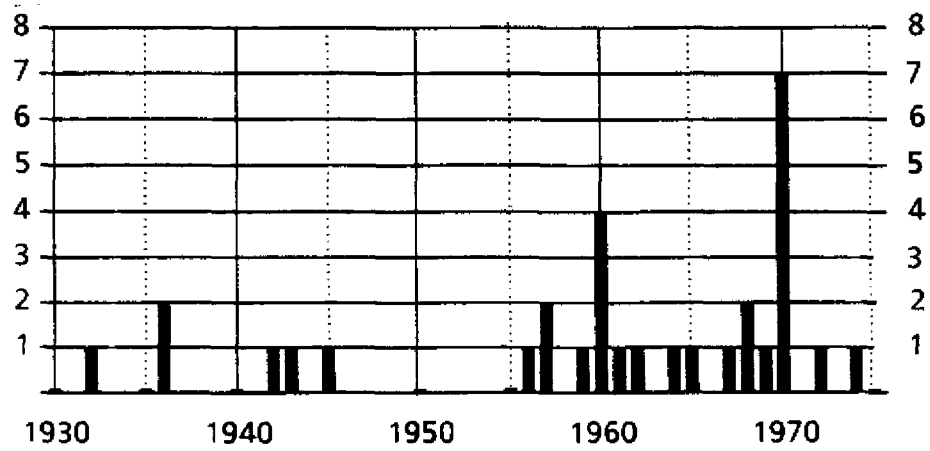


FIGURE 1. NUMBER OF STATIONS SHOWING THE MOST PROBABLE BREAK YEAR FOR CHANGE IN MEAN ANNUAL RAINFALL.

D. Discussion

The purpose of this analysis is to detect the time of change of mean annual rainfall due to large scale, natural systems shift, or to man-made impacts. In this sense, the change sought shall be regional, not local, around the individual gaging stations. The method used, however, is not able to differentiate the source of changes whether due to the regional processes; due to the change of instrumentation; or due to change of the local environment close to the gaging stations. It is argued that if the change occurred only at the gaging stations, it would not show in the regional scale, i.e. the change would not have a regional trend. Figure 1 shows that the majority of possible changes occurred between 1960 and 1970; this is believed to indicate that a change in the regional scale might have occurred around these years. The next analysis is to see if the amount of change at these points is statistically significant.

IV. DETECTING THE AMOUNT OF CHANGE IN THE MEAN OF MONTHLY, SEASONAL, AND YEARLY RAINFALL

A. Data Treatment

The same set of data used in Section III is used here. The missing data position was kept in the analysis. Only 33 long term stations have records after 1975.

B. Method of Analysis

Computer programs for "t" and "F" tests are available from the same Lecture Notes. Given a time series of data, mean and variance tests are done at several break points of the series. For example, given total annual rainfall of a station from 1914 to 1981, with break point at 1970, two samples are formed: Sample 1 from 1914 to 1969, and Sample 2 from 1970 to 1981. "t" and "F" tests are done on these two samples to see if they are significantly different. Confidence level is set at 90%.

C. Results

In general, rainfall has decreased in recent years. The amount of decrease, however, may not be statistically significant at all the stations. Figures 2, 3, and 4 are plots from an average of all the stations to show the District-wide trend. On an annual basis, average annual rainfall decreased about 9% (5 inches) for the period after 1970, as compared to the prior period. Most of the decrease comes from drier wet seasons. Wet season rainfall decreased about 4.3 inches (10.7%) after 1970. Wet season is defined as May through October inclusive. Undulations in Figures 2 and 3 indicate that a cyclic trend of some sort may be existing. Figure 4 shows that recently wet seasons were drier and shorter; while dry seasons were wetter. The inversion between October and

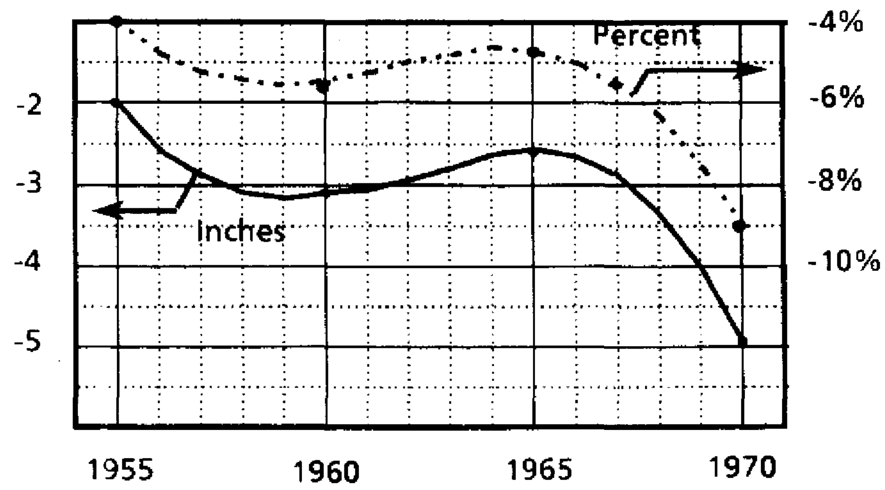


FIGURE 2a. DIFFERENCE BETWEEN AVERAGE ANNUAL RAINFALL FOR THE PERIODS BEFORE AND AFTER A GIVEN BREAK YEAR.

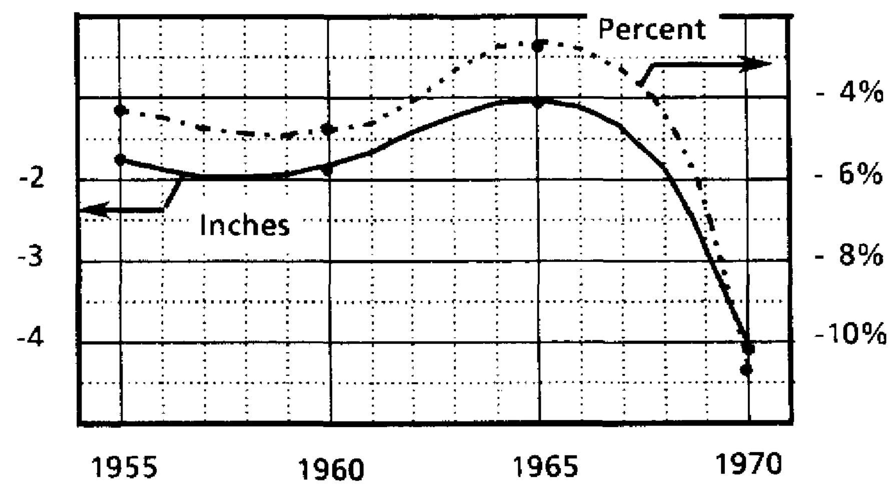


FIGURE 2b. DIFFERENCE BETWEEN AVERAGE WET SEASON RAINFALL FOR THE PERIODS BEFORE AND AFTER A GIVEN BREAK YEAR.

November is very interesting. The causes and implications of this inversion require further studies.

It is not too difficult to understand the impact of a decrease in rainfall on resource management. Of similar importance is the rainfall distribution. As shown in Figure 4, wetter dry seasons

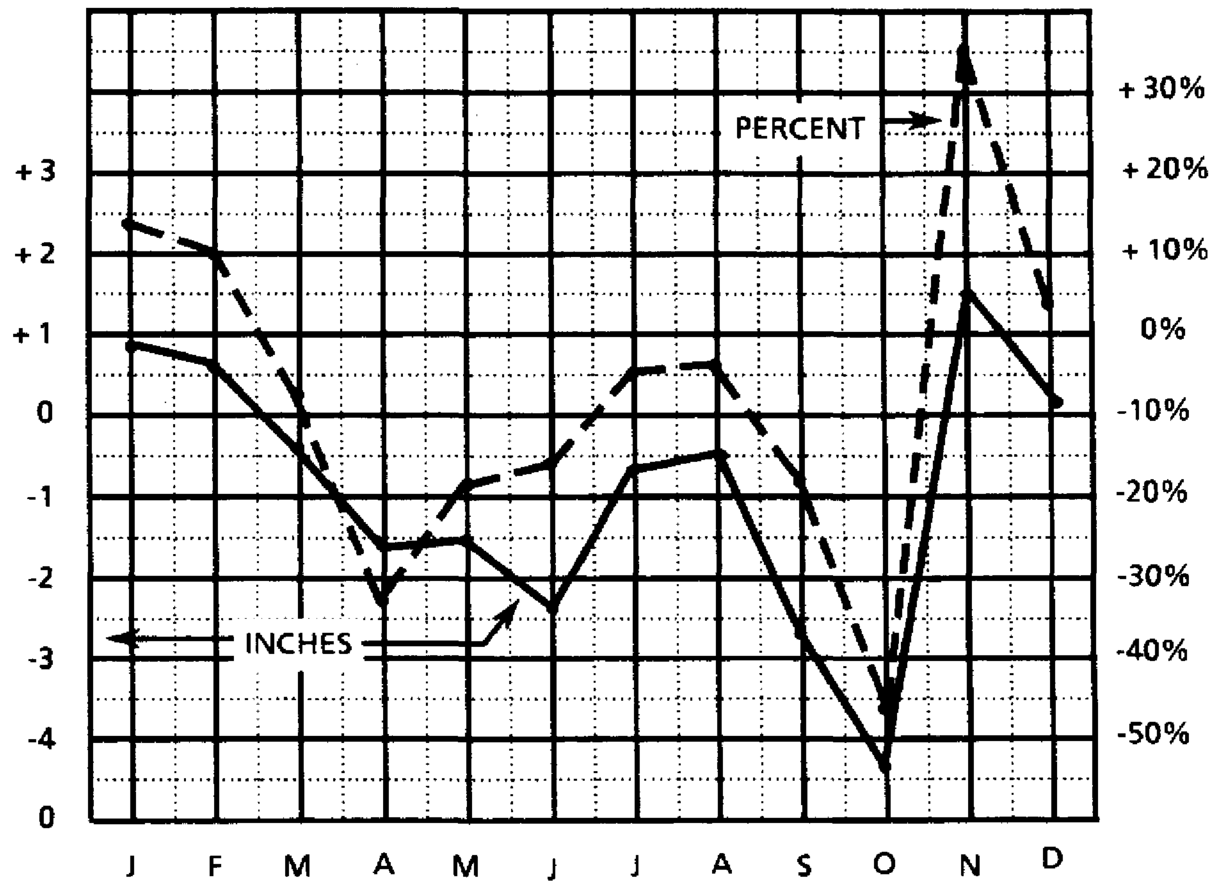


FIGURE 3. DIFFERENCE IN AVERAGE MONTHLY RAINFALL FOR THE PERIODS BEFORE AND AFTER YEAR 1970.

and drier wet seasons after 1970 mean that rainfall occurred more uniformly throughout those years. In other words, the difference between wet season and dry season rainfall is decreasing. Specifically, before 1970, wet season minus dry season rainfall was about 24.9 inches; after 1970, it was 21.9 inches, a reduction of about 3 inches, or 12%. Extending the "reservoir" replenish period, it can be viewed that a wet season is sandwiched by two dry seasons, or two wet seasons sandwich a dry season. It is found that there was almost a 20% (2.2 inches) reduction

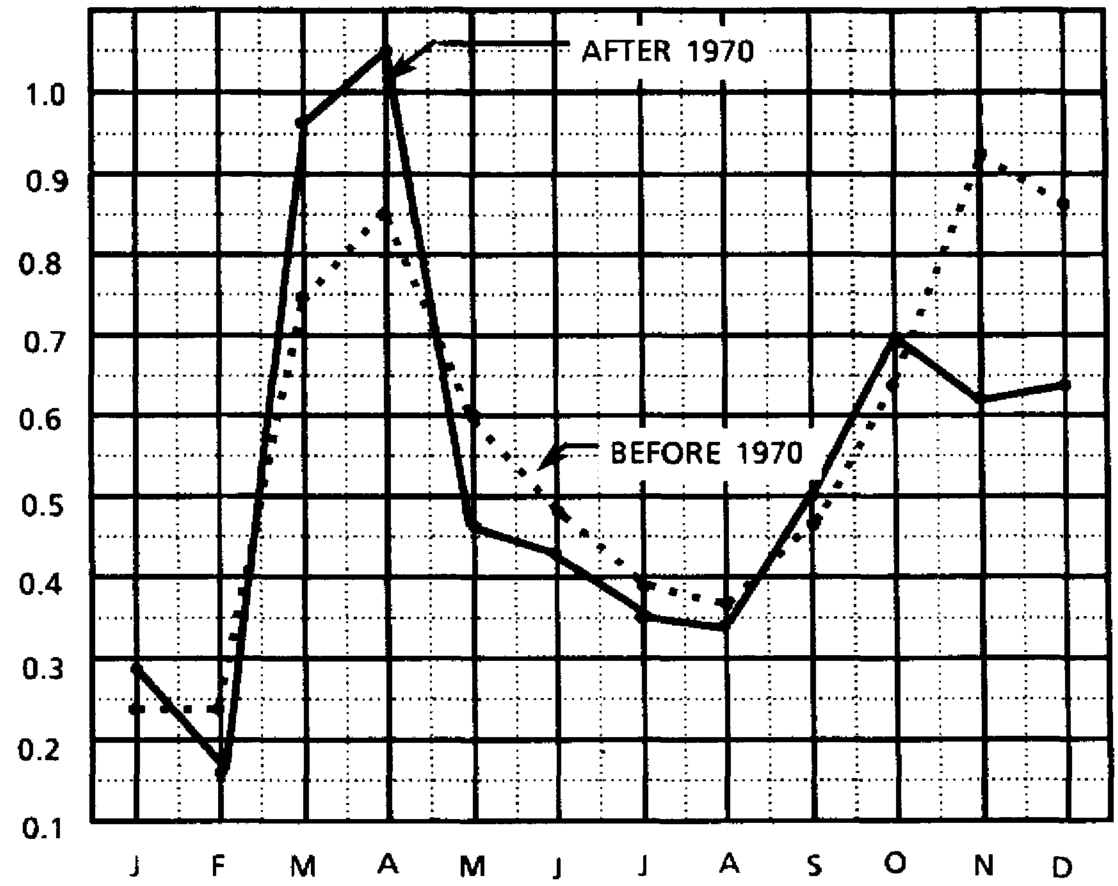


FIGURE 4. COEFFICIENT OF VARIATION OF EACH MONTH FOR THE PERIODS BEFORE AND AFTER 1970.

in the difference of the middle wet season to replenish the flanking two dry seasons after 1970.

For frequency analysis, change of the variance may be more important than change of the mean. From a statistical viewpoint, however, there was little change in the variance. In general, the variances after 1970 were even less than those before 1970. Figure 4 shows the change of variance in terms of variation coefficients. Note the drastic changes in the variation for months bordering wet seasons.

V. DETECTING THE AMOUNT OF CHANGE IN THE MEAN OF DAILY RAINFALL PARAMETERS

A. Data Treatment

All the daily rainfall data available in the data base (with record length over 50 years) were used for the analysis. Since most applications of short duration rainfall analyses are in flood control, high intensity rainfall parameters are sought after. These parameters are accumulated on a yearly basis, hence the position of missing daily data is not important. For simplicity, missing daily data were ignored. There are 21 stations with 50 or more years of daily record that lasted beyond 1975.

B. Method of Analysis

The same methods used in monthly and yearly rainfall analysis are used here. Break points are 1955, 1960, 1965, and 1970.

C. Results

1. Contribution of heavy storms to the total rainfall.

   It was observed that fewer hurricanes visited south Florida since the establishment of the Central and Southern Florida Flood Control District. Hurricanes and tropical depressions are usually accompanied by heavy rainfall. This analysis is intended to quantify the amount of heavy rainfall contribution to the total rainfall and the change of the contribution, if any. Figure 5 shows that storms with rainfall over 1 in/day contributed about 27 in/year, or close to one-half of the total rainfall in a year. Figure 5 also shows that heavy storms have decreased steadily since 1955. Figure 6 shows that the average total events of heavy rainfall per year also decrease correspondingly. Note that these figures are obtained by averaging all the stations together, so there are fractions in

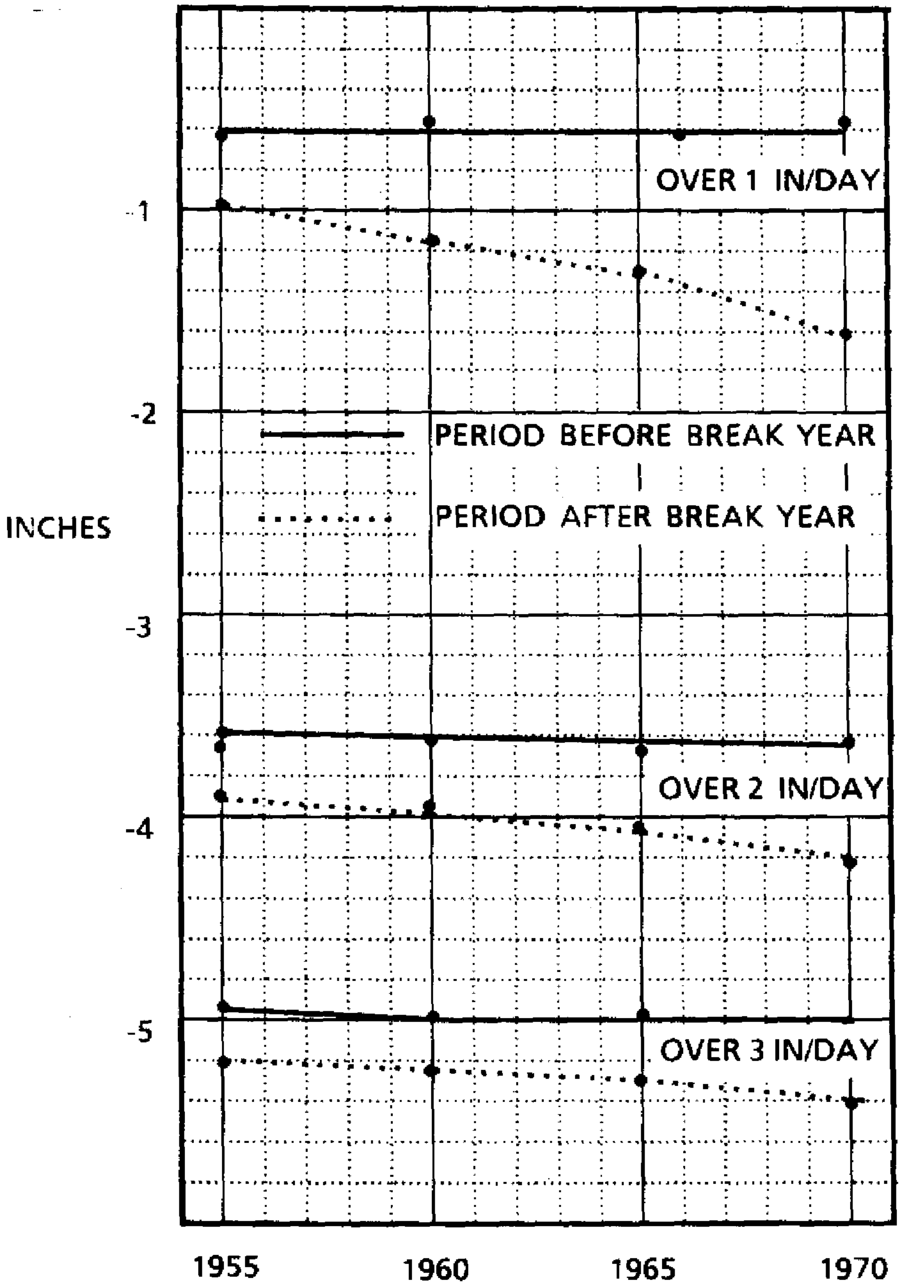


FIGURE 5. AVERAGE TOTAL AMOUNT OF HEAVY RAINFALL PER YEAR BEFORE AND AFTER A GIVEN BREAK YEAR

the number of events. Figure 7 (curve a) shows that the contribution of heavy rainfall decreases fairly uniformly as

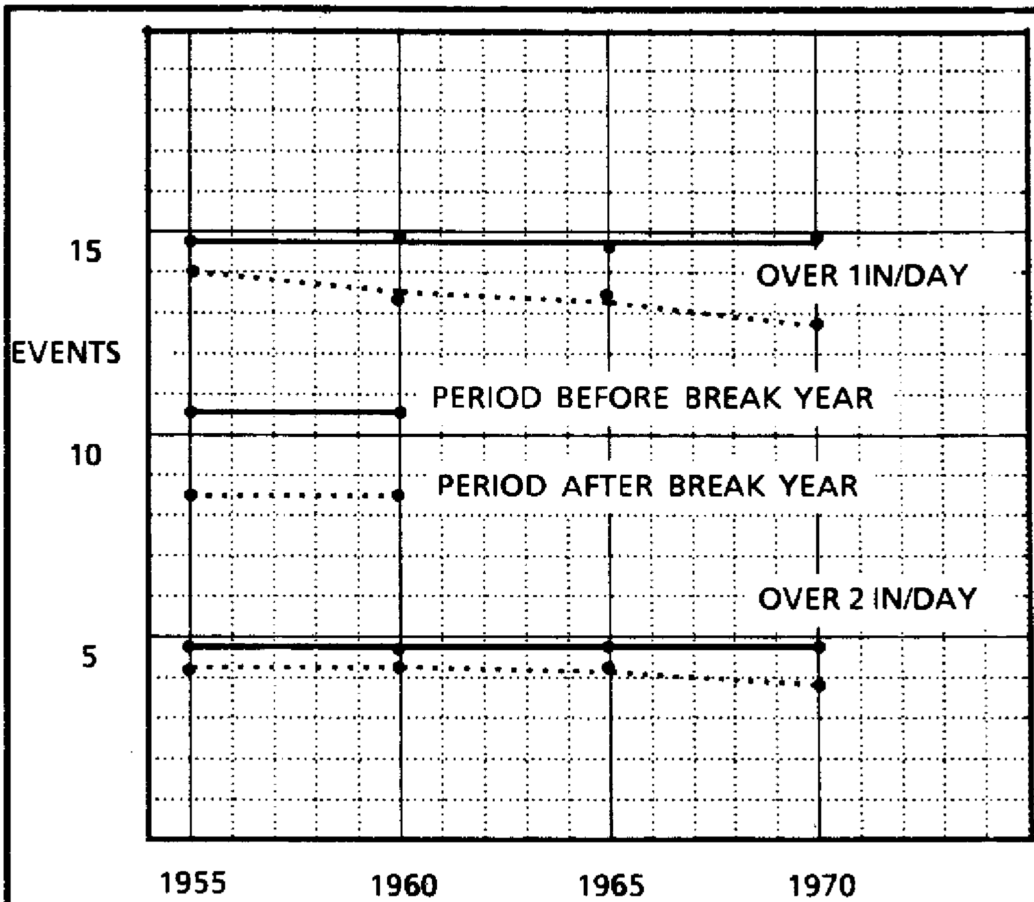


FIGURE 6 AVERAGE TOTAL EVENTS OF HEAVY RAINFALL PER YEAR BEFORE AND AFTER GIVEN BREAK YEAR

rainfall intensities increase; while curve b indicates that high intensity rainfall contribution decreases after 1970 much more than low intensity rainfall. Also, from Figures 2, 5, and 7c, one can see that most of the annual rainfall decrease was accounted for by decreases in heavy rainfall.

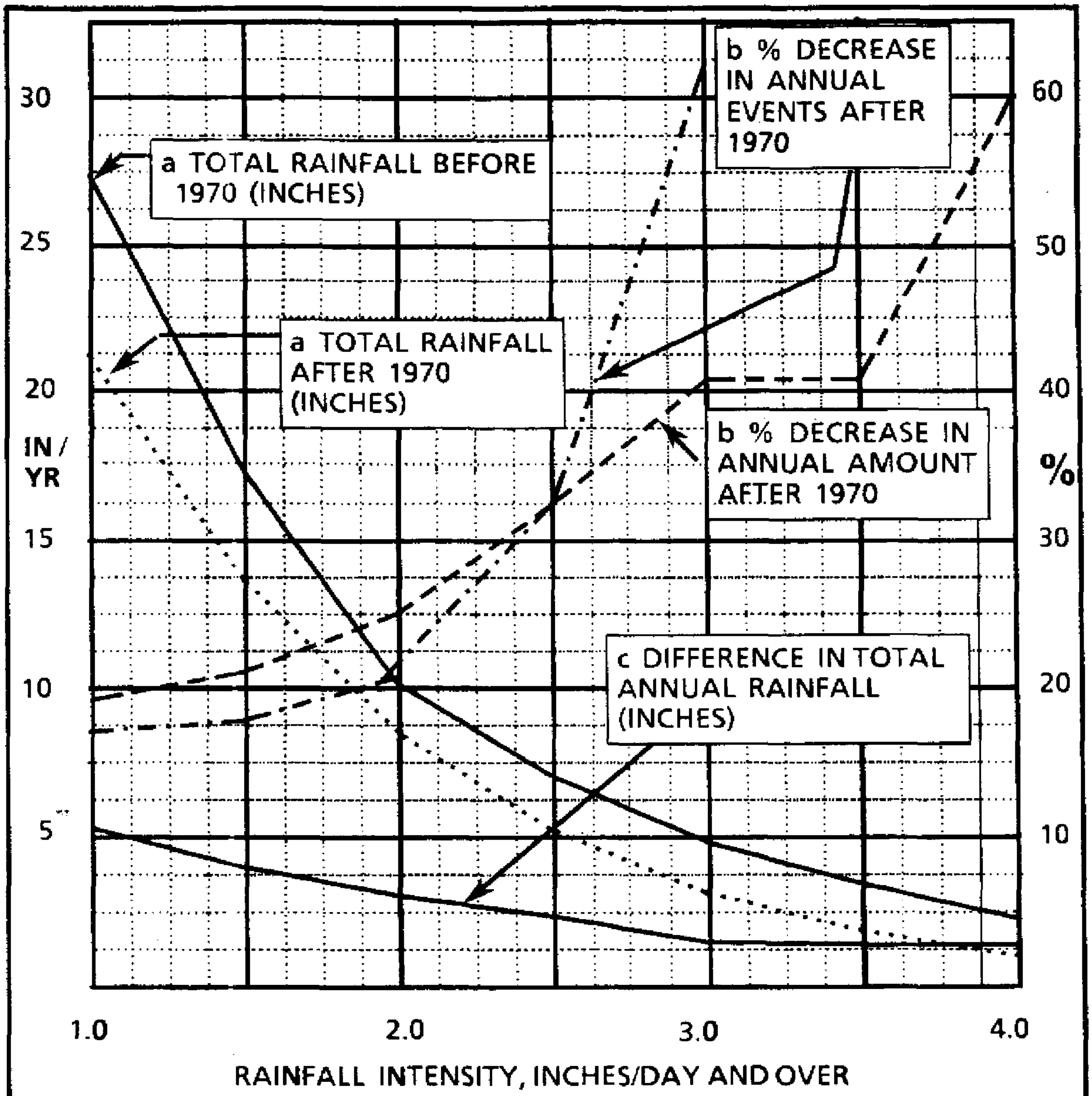


FIGURE 7 HIGH INTENSITY RAINFALL BEFORE AND AFTER 1970

It is difficult to differentiate tropical storm rainfall and local thunderstorm rainfall on the basis of rain gage data. Brandes (2) indicated that hurricanes and tropical storms contributed an average of 3.79 in/year (p.50); and each hurricane or tropical storm contributed an average rainfall of

2.68 inches (p.102). The data (Table 4, p.51) also showed that prior to 1969 there were 4.24 inches/year of rainfall caused by tropical cyclones, while there were only 1.56 inches/year of tropical cyclone rainfall after 1969. This reduction of 2.68 inches/year (4.24 inches-1.56 inches = 2.68 inches) from tropical cyclone rainfall accounted for 54% of the 5 inch annual rainfall reduction after 1970. It is also interesting to point out the reduction of high intensity rainfall as shown in Figure 7, curve c. For rates over 2.0 inches/day the decrease was 2.48 inch/year. This indicates that most of the high intensity rainfall reduction is due to decreasing tropical cyclone rainfall. Figure 8, however, shows that variance of year to year heavy rainfall increases, which implies that heavy rainfall recurrence intervals may not increase at all.

2. Change of parameters in annual series of daily rainfall.
   Annual series of daily rainfall has been used for rainfall frequency analysis. Usually the following equation is used:

$$Yt = m(1 + Cv.Kt)$$

where

Yt = magnitude (of rainfall) at a recurrence interval of t year,

m = mean,

Cv = variation coefficient (Note that standard deviation equals the product of mean and Cv), and

Kt = coefficient for t recurrence year. Kt depends on type of distribution used. Kt can be found in tables for different distributions.

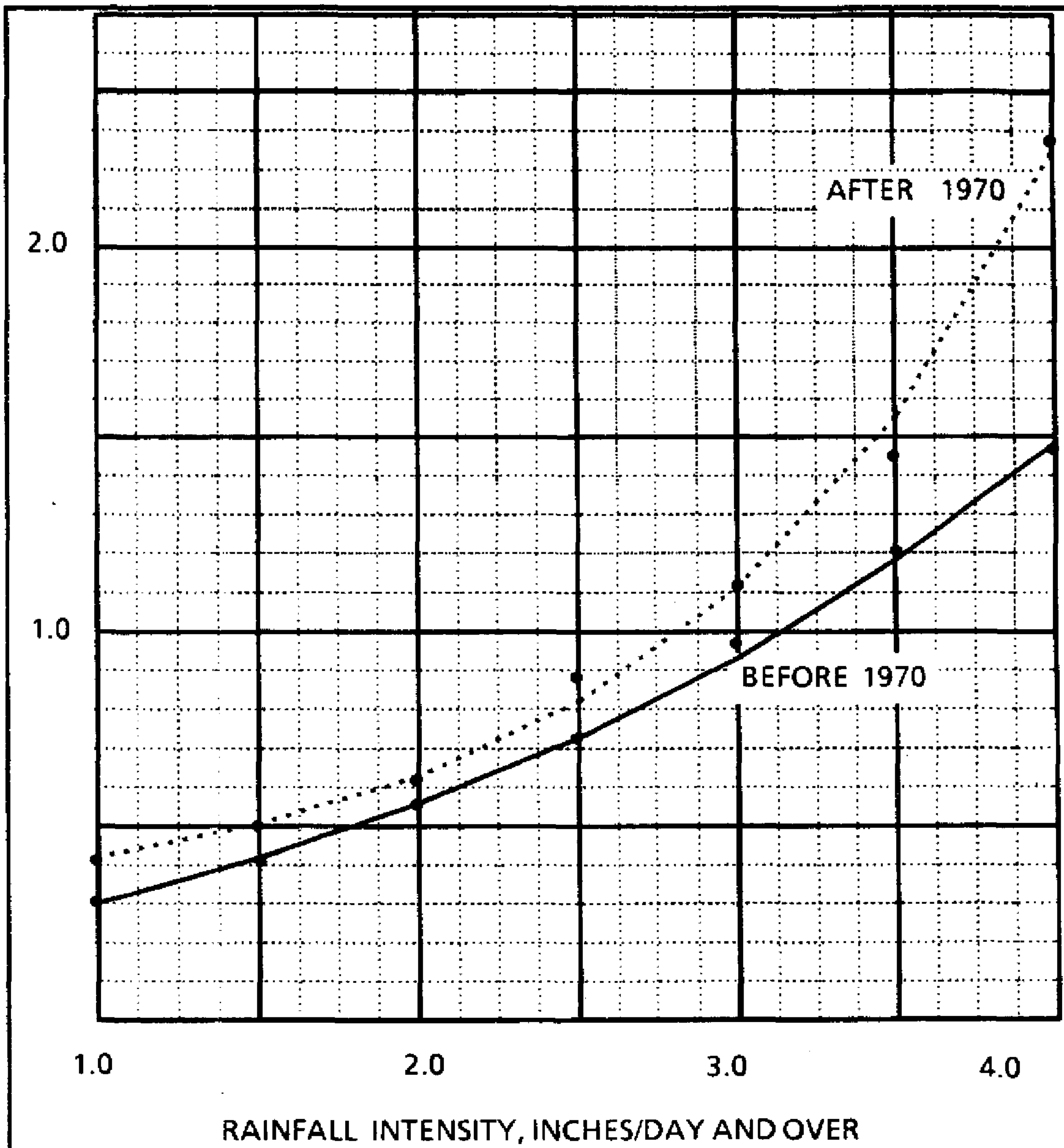


FIGURE 8 COEFFICIENTS OF VARIATION OF TOTAL YEARLY RAINFALL AT GIVEN INTENSITY AND OVER, IN ANNUAL SERIES BEFORE AND AFTER 1970.

Changes of mean and variation coefficient in the annual series may have an important implication in flood control operations. Figure 9 sums up the findings in the average sense. Note that a sharp increase in coefficient of variation occurred after 1970. Eleven (11) out of 21 stations showed a significant change in the variance before and after 1970. From the above equation, it is obvious to see that Yt will increase proportionally as Cv increases.

VI. LOCATION OF CHANGE

A. Data Treatment

Those data obtained in the previous analyses were plotted on maps in the hope of gaining some insight into the spatial distribution of the changes.

B. Method of Analysis

Contour maps are made from data points by a computer generated, hand smoothed method. It should be cautioned that this is not a regional analysis method, hence the values interpreted from these maps should not be taken quantitatively without qualification. Suppose a value f is read from one of these maps, it means that for recording station(s), if any, in this area, the f value has been derived from the records of the individual station(s).

C. Results

Basically there are two sets of maps. One set shows the quantity of changes, and the other set shows the statistical significance of the changes. The quantities of change are self-explanatory in the maps. The statistical significance is tested at 90% confidence level. Approximate value of the stations at this level for the sample size (degrees of freedom) is specified in the overall sense.

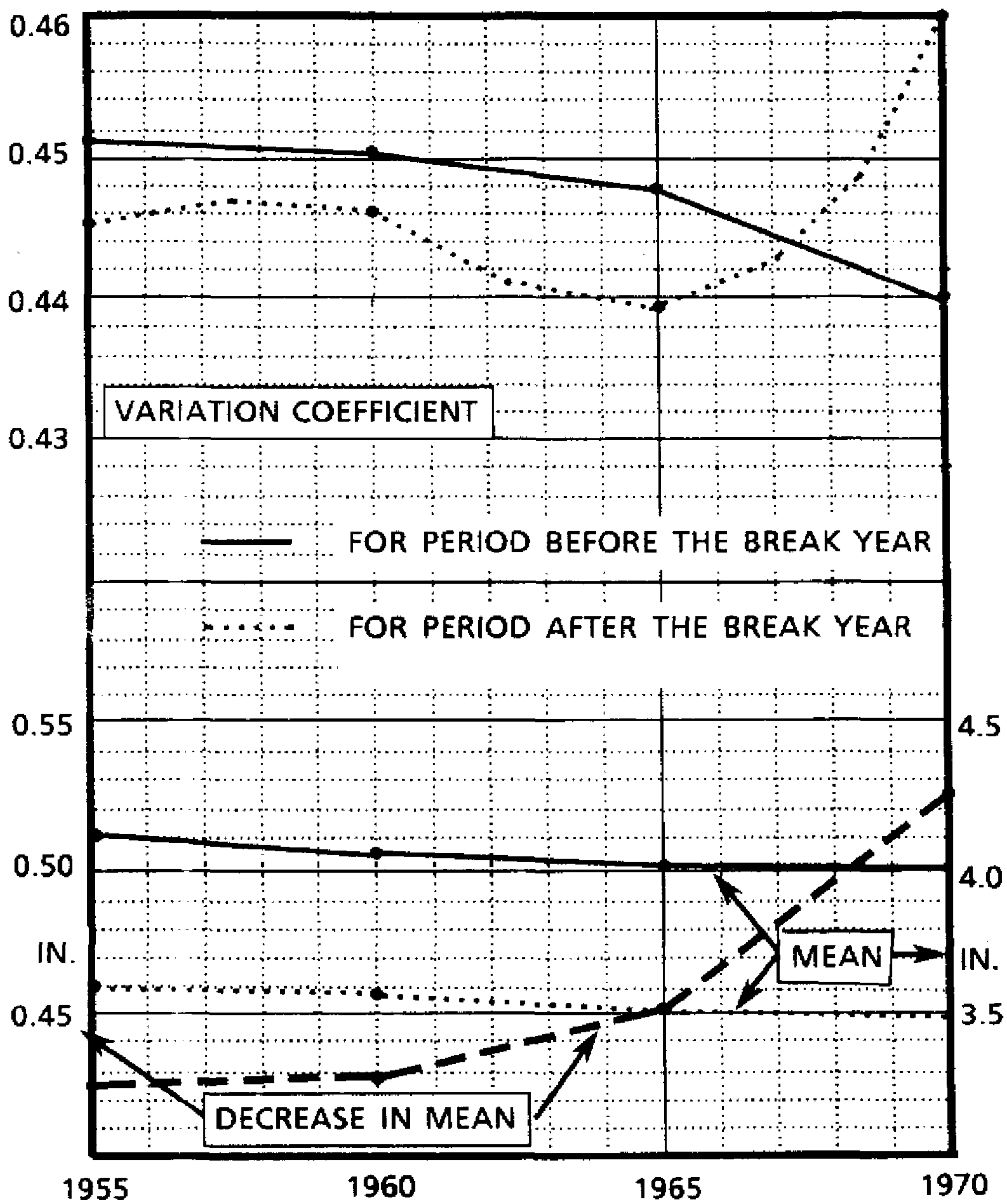


FIGURE 9 MEAN AND VARIATION COEFFICIENT IN ANNUAL SERIES

For example, in t test, t greater than 1.64 indicates a significance at 90% confidence with infinite degrees of freedom. Most stations, however, have about 50 degrees of freedom and require t greater than 1.68 to be significant at the 90% level. Note that contours are not plotted at equal intervals to reduce lines on the maps.

These two sets of maps are organized into three groups. The first group, Figures 10 through 17, deals with monthly and yearly rainfall. It shows that rainfall records in the Kissimmee River and Hillsboro Canal have a significant decrease in rainfall after 1970. The second group of maps, Figures 18 through 23, shows the change of yearly heavy rainfall. The change occurred mostly in stations around the Kissimmee River and the southwest corner of the District. The third group, Figures 24 through 27, showing changes in means and variation coefficients in the annual series of daily rainfall, is less consistent between the magnitude of change and the significance of change. This is expected because the previous two groups of maps are constructed from data accumulated through a period of time which has a smoothing effect, while this group of maps is constructed from extreme data of short-time step which is opposite to smoothing. Furthermore, because of the contouring technique used and the wide range of computed F values from 1.24 to 10.16, the mapped F values tend to be high. This is why Figure 27 shows that most of the District areas have F greater than 2.5. Even discounting the reliability of Figure 27, the change of variation coefficients in many areas, as shown in Figure 26, still can not be ignored.

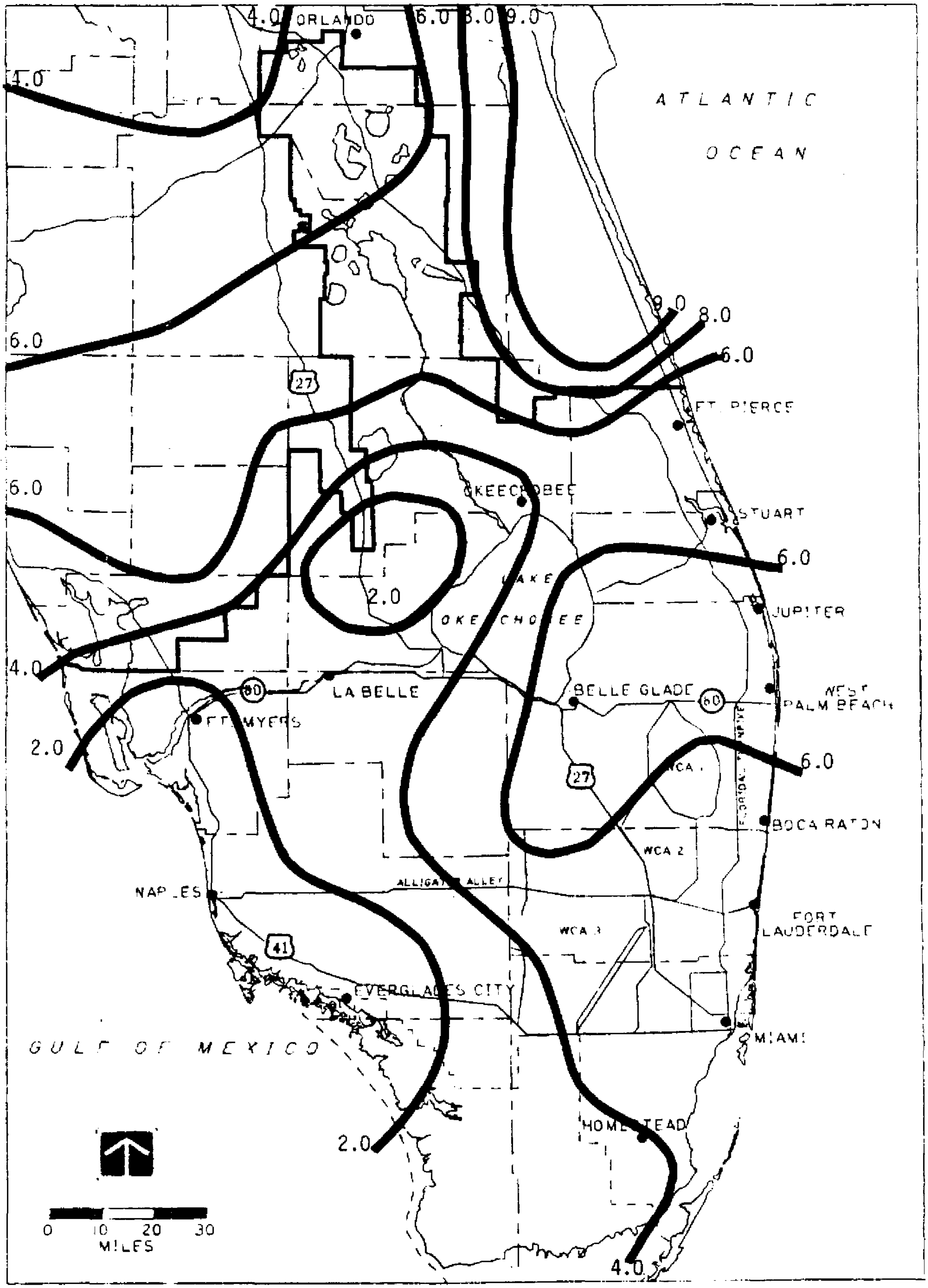


FIGURE 10. ANNUAL RAINFALL DECREASE, INCHES. BREAK AT 1970.

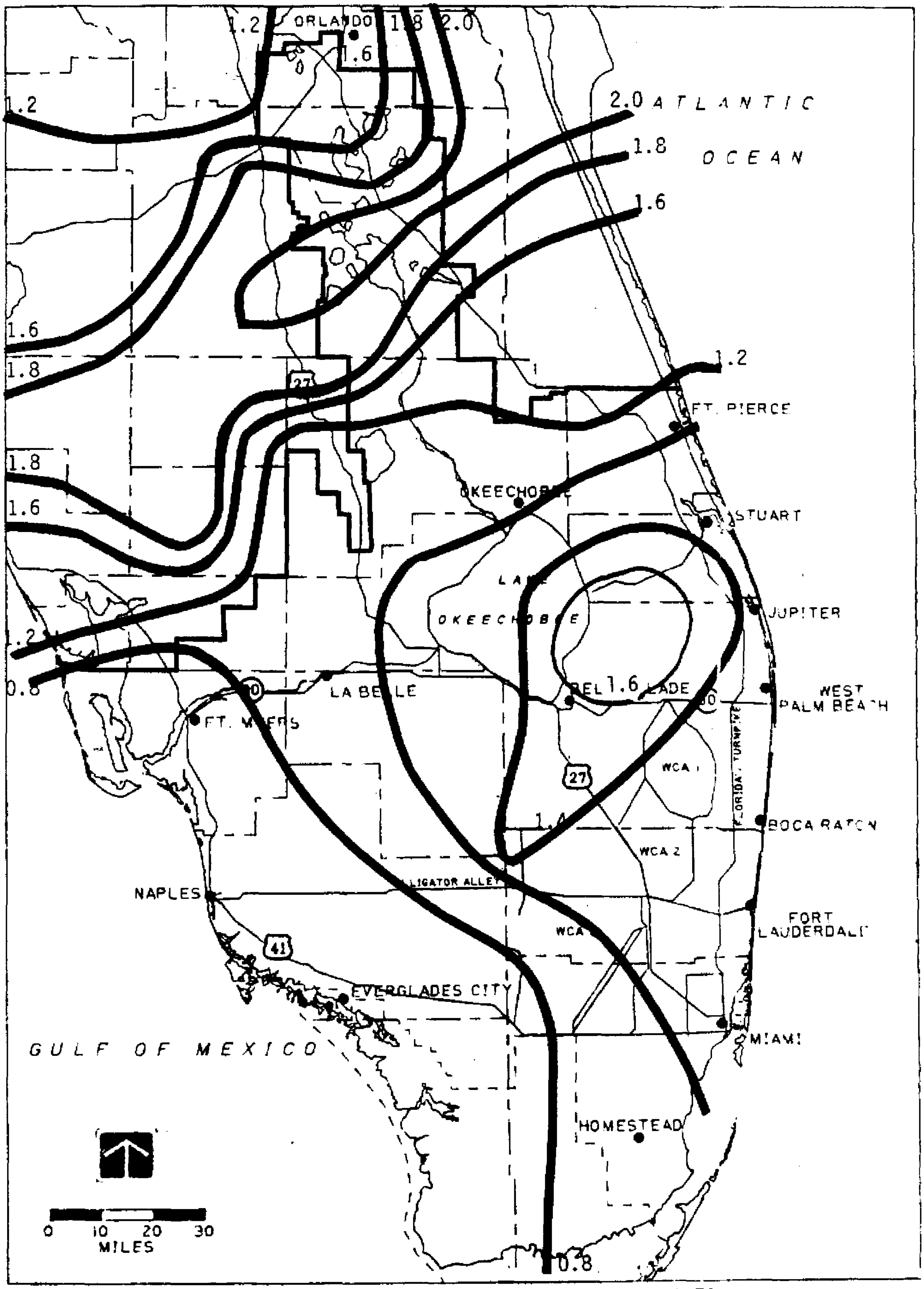


FIGURE 11. t VALUES FOR ANNUAL RAINFALL CHANGE. BREAK AT 1970.
t>1.6 INDICATES A SIGNIFICANT CHANGE OF MEAN AT 90% CONFIDENCE LEVEL.

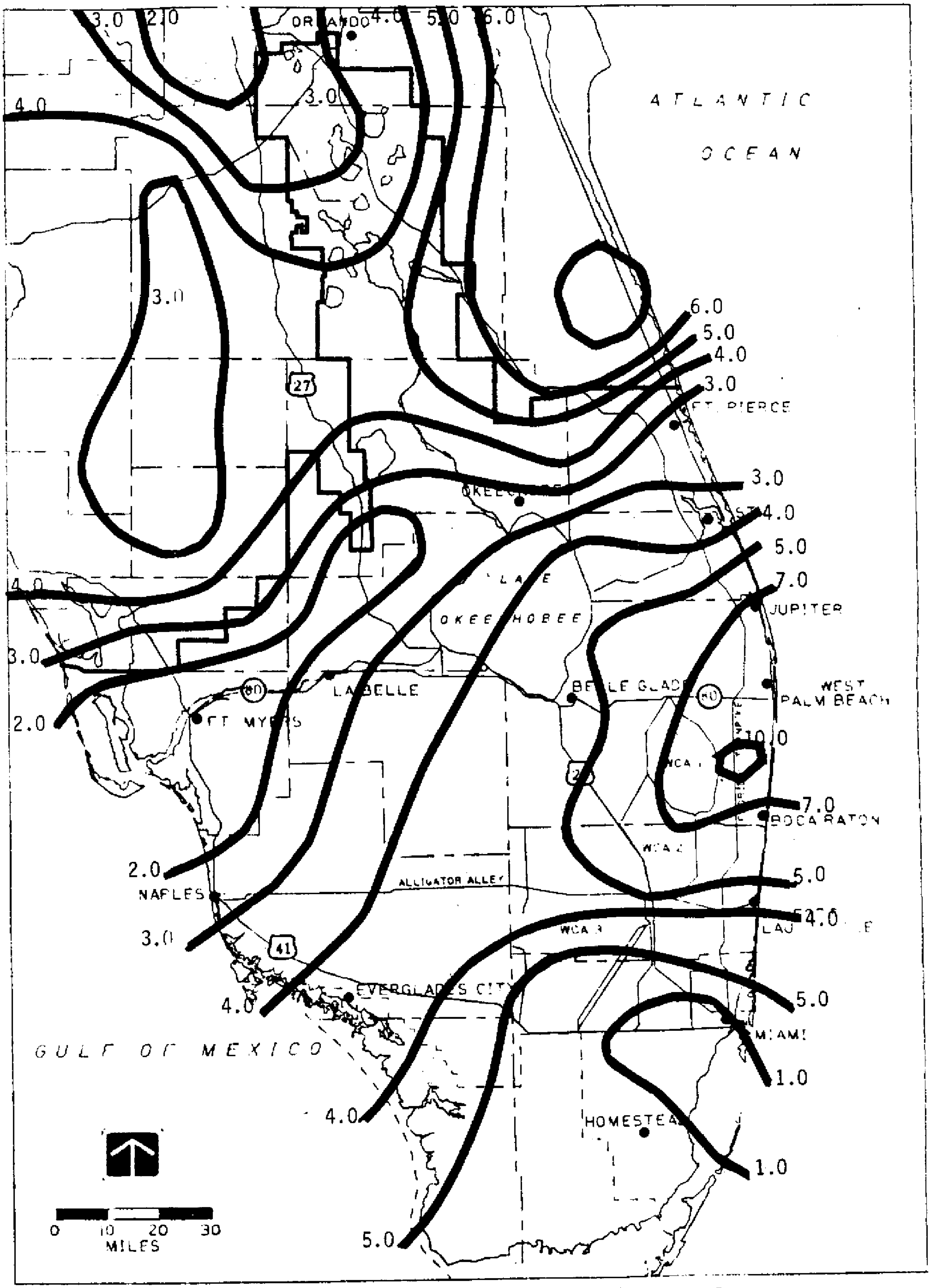


FIGURE 12. WET SEASON RAINFALL DECREASE, INCHES, BREAK AT 1970.

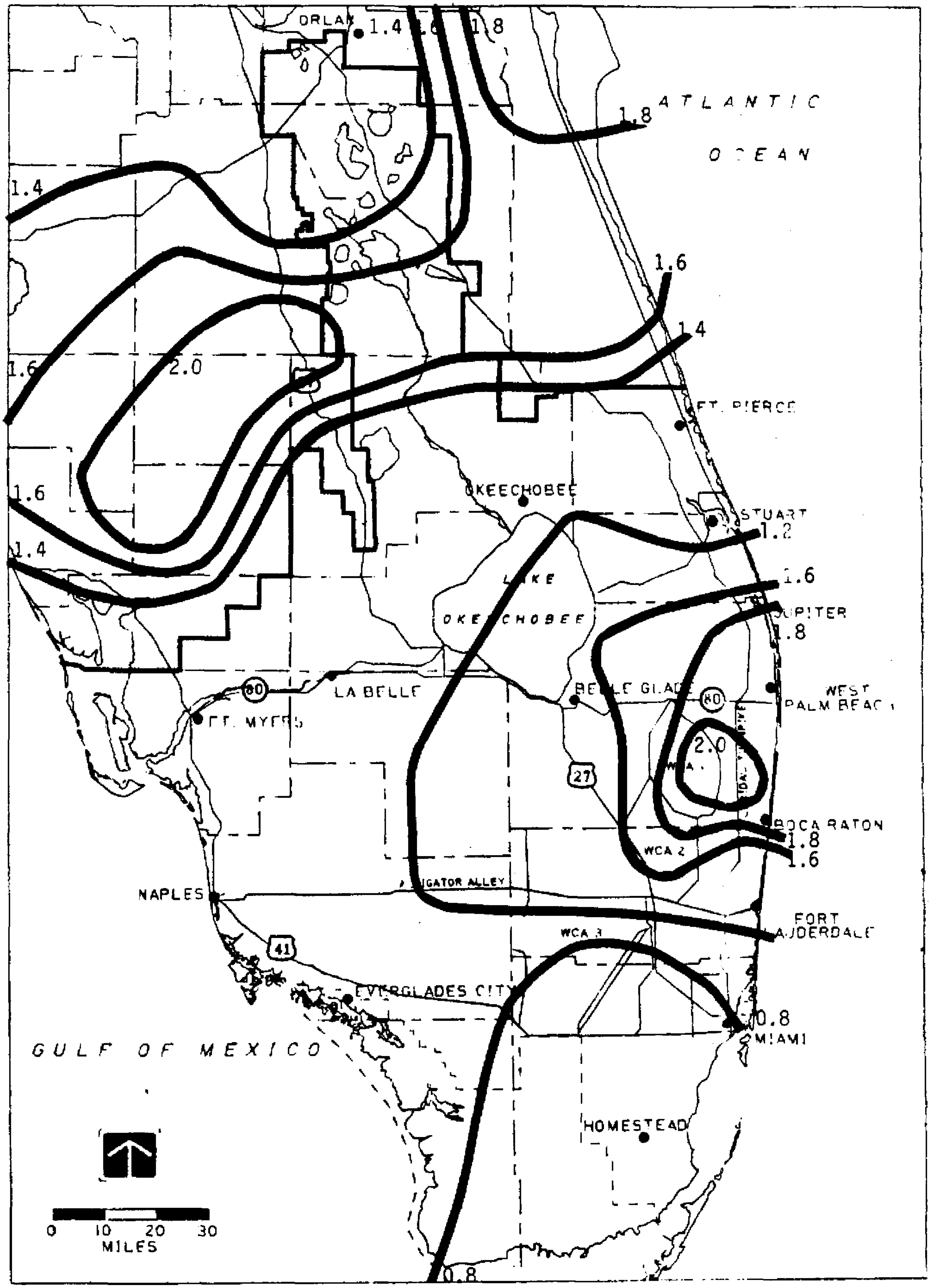


FIGURE 13. t VALUES FOR WET SEASON RAINFALL CHANGE, BREAK AT 1970. t>1.6 INDICATES A SIGNIFICANT CHANGE IF MEAN AT 90 % CONFIDENCE LEVEL.

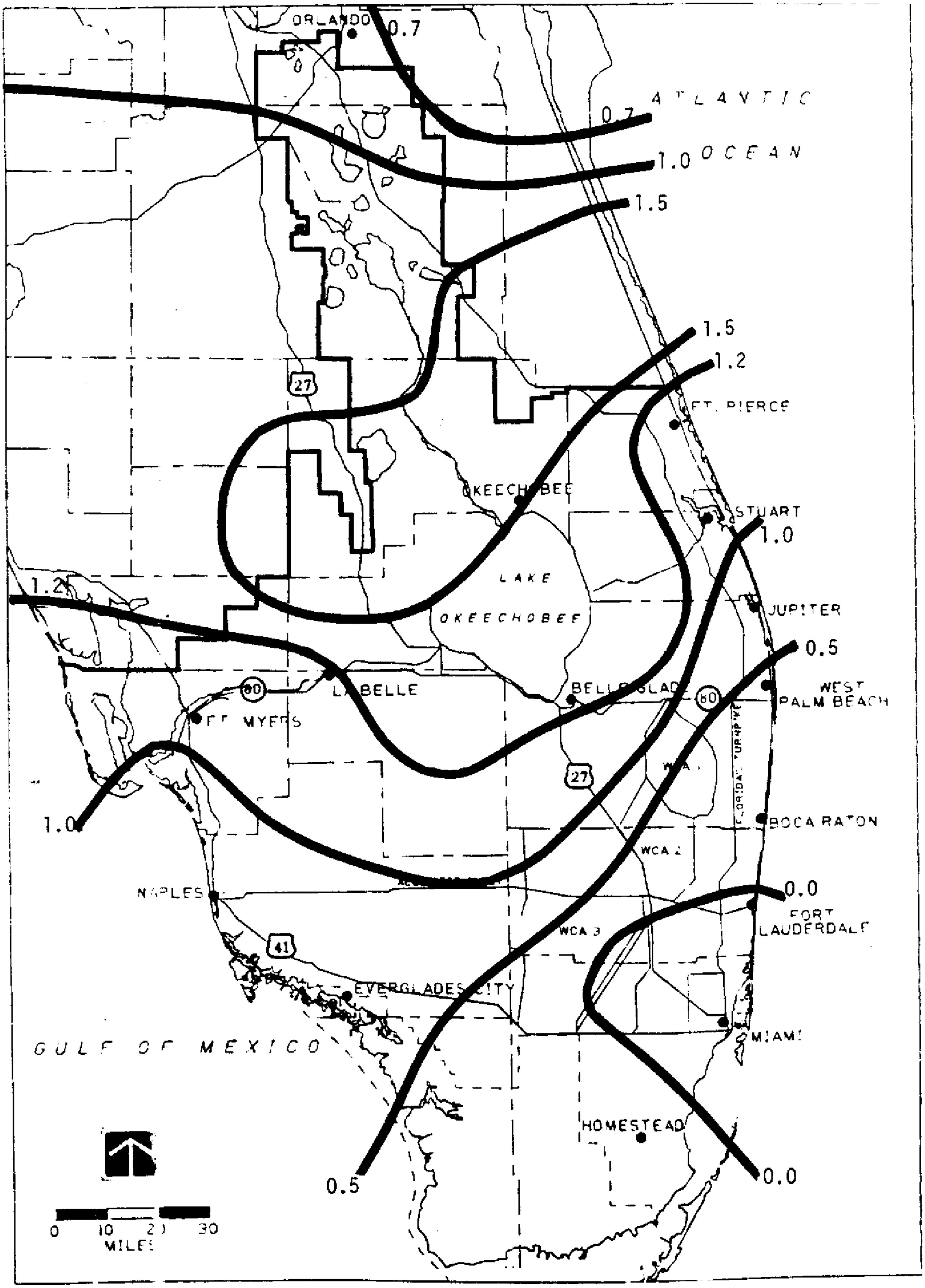


FIGURE 14. APRIL RAINFALL DECREASE. INCHES. BREAK AT 1970

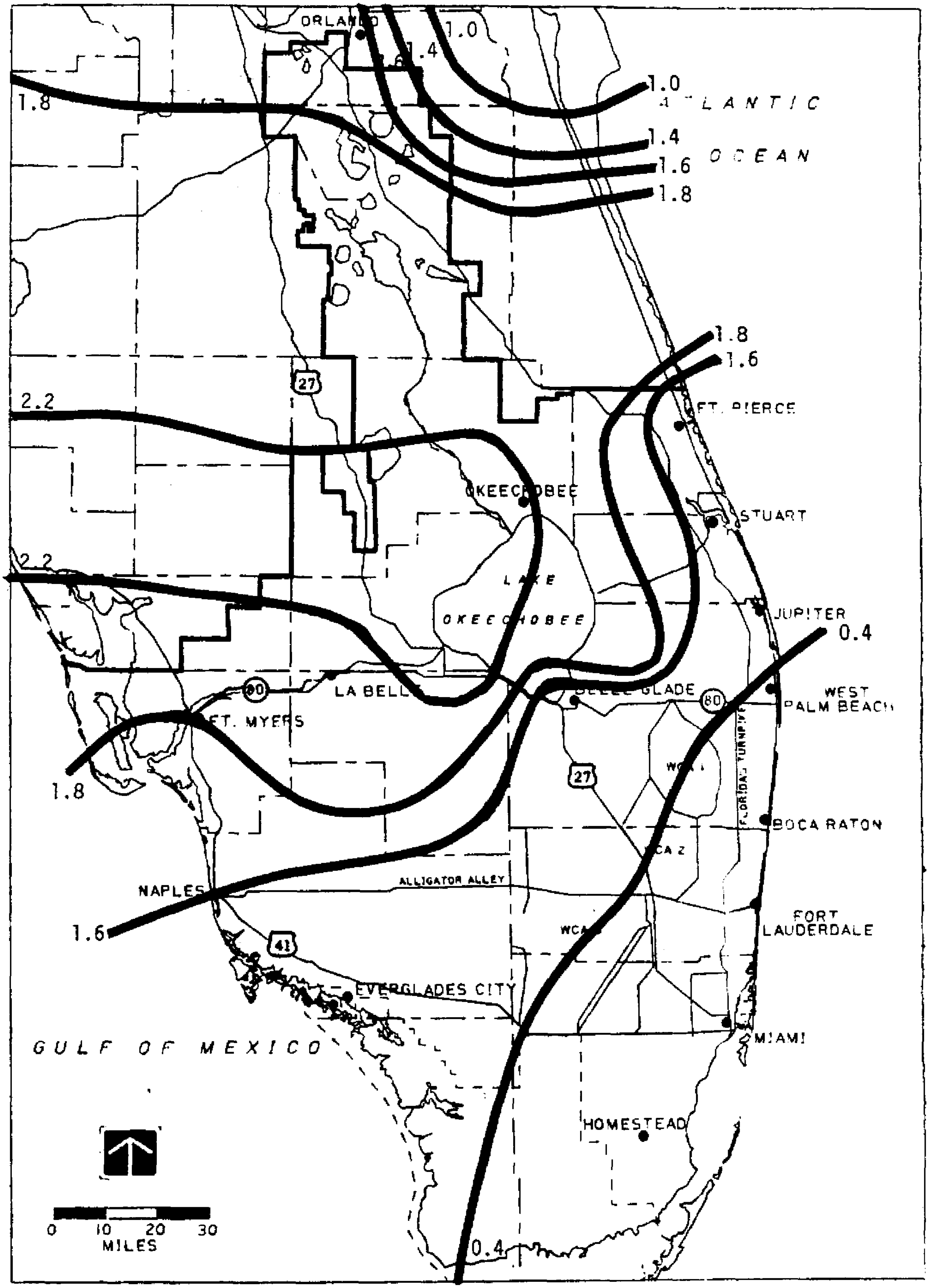


FIGURE 15. t VALUES FOR APRIL RAINFALL CHANGE, BREAK AT 1970
$t>1.6$ INDICATES A SIGNIFICANT CHANGE AT 90% CONFIDENCE LEVEL.

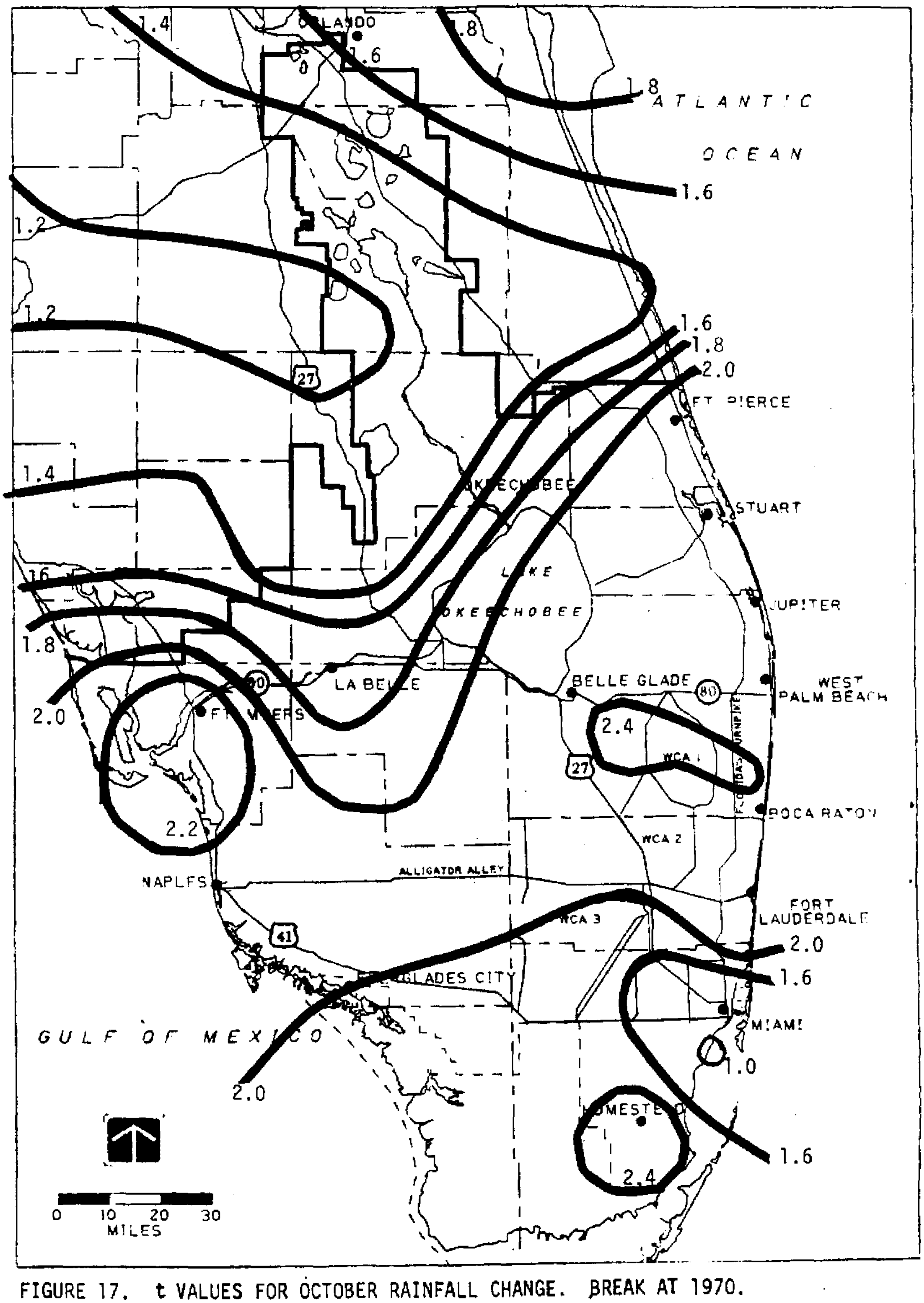


FIGURE 17. t VALUES FOR OCTOBER RAINFALL CHANGE. BREAK AT 1970.
t>1.6 INDICATES A SIGNIFICANT CHANGE AT 90% CONFIDENCE LEVEL

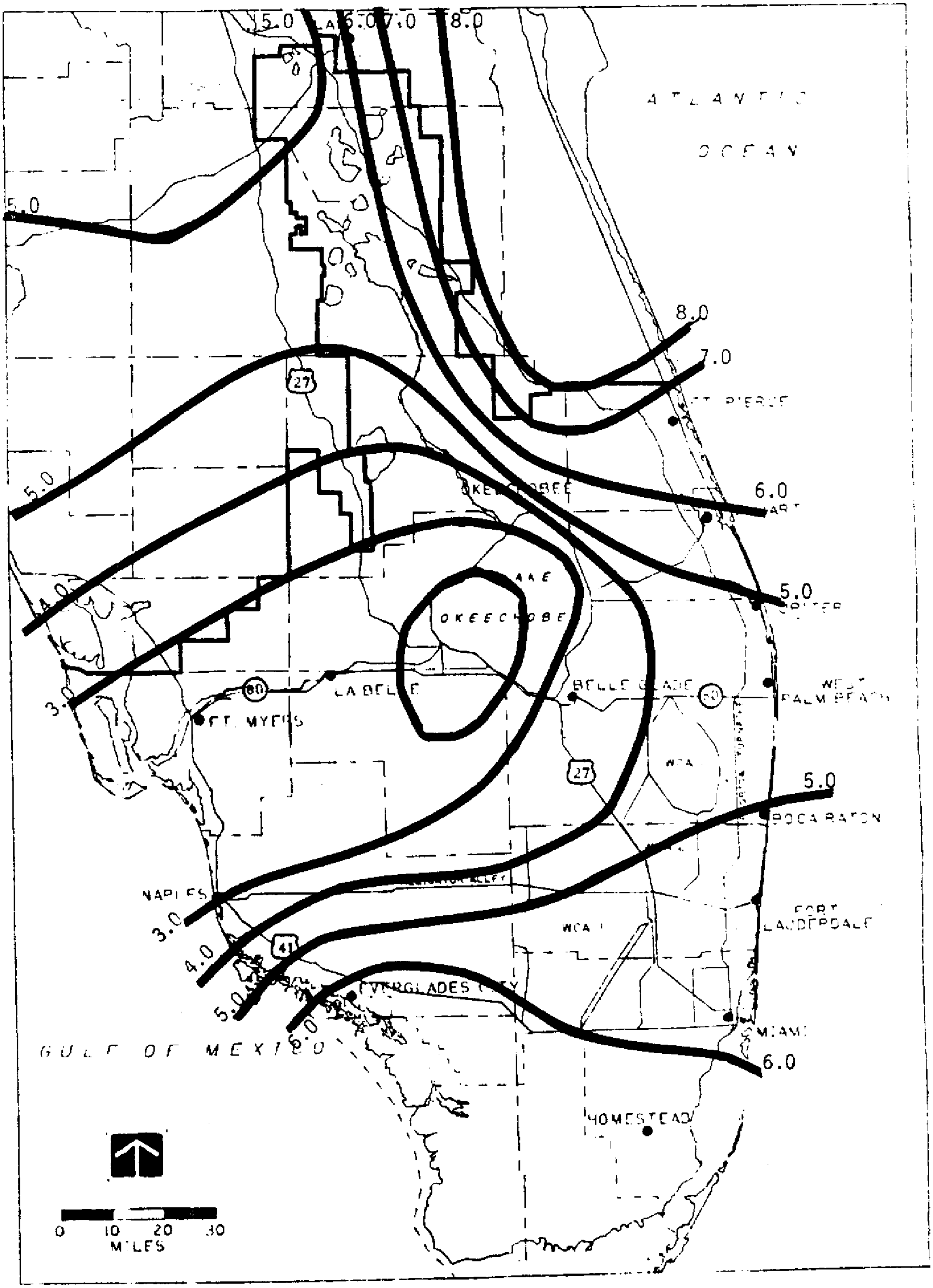


FIGURE 18. YEARLY MEAN DECREASE OF OVER 1 IN/DAY RAINFALL, INCHES. BREAK AT 1970.

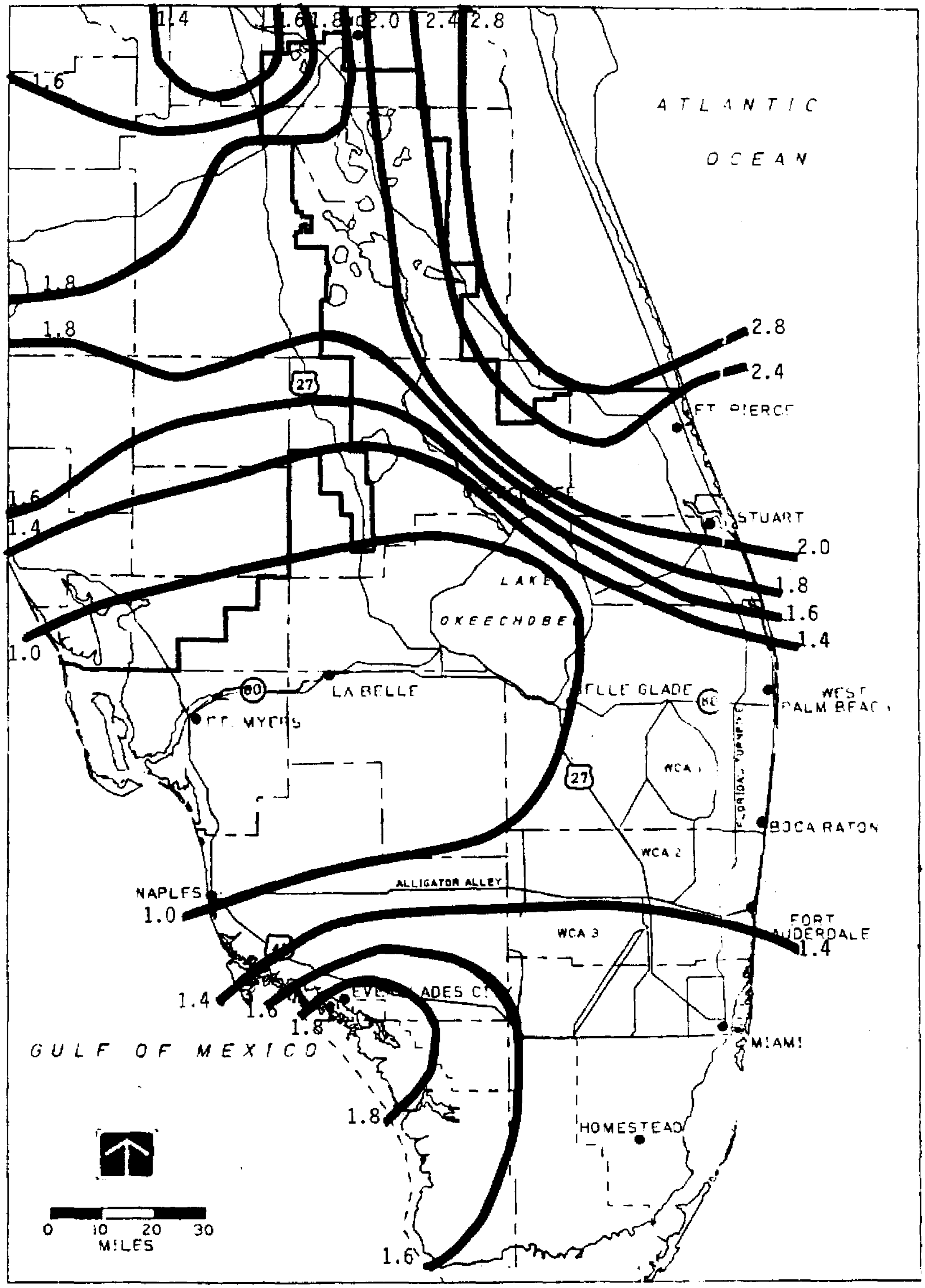


FIGURE 19. t VALUES FOR OVER 1 IN/DAY RAINFALL CHANGE. BREAK AT 1970.
t>1.6 INDICATES A SIGNIFICANT CHANGE OF MEAN AT 90% CONFIDENCE LEVEL.

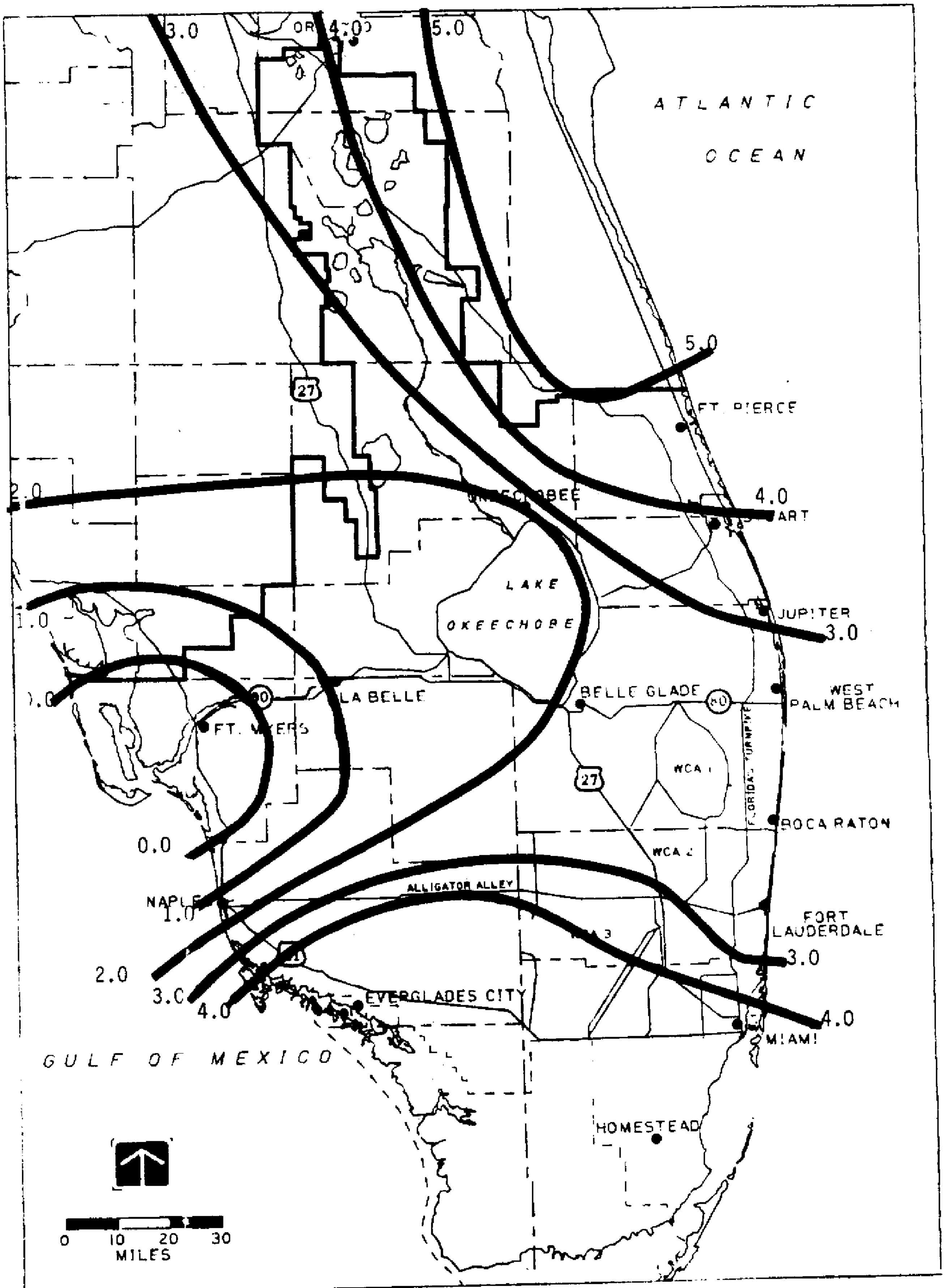


FIGURE 20. YEARLY MEAN DECREASE OF OVER 2 IN/DAY RAINFALL, INCHES. BREAK AT 1970.

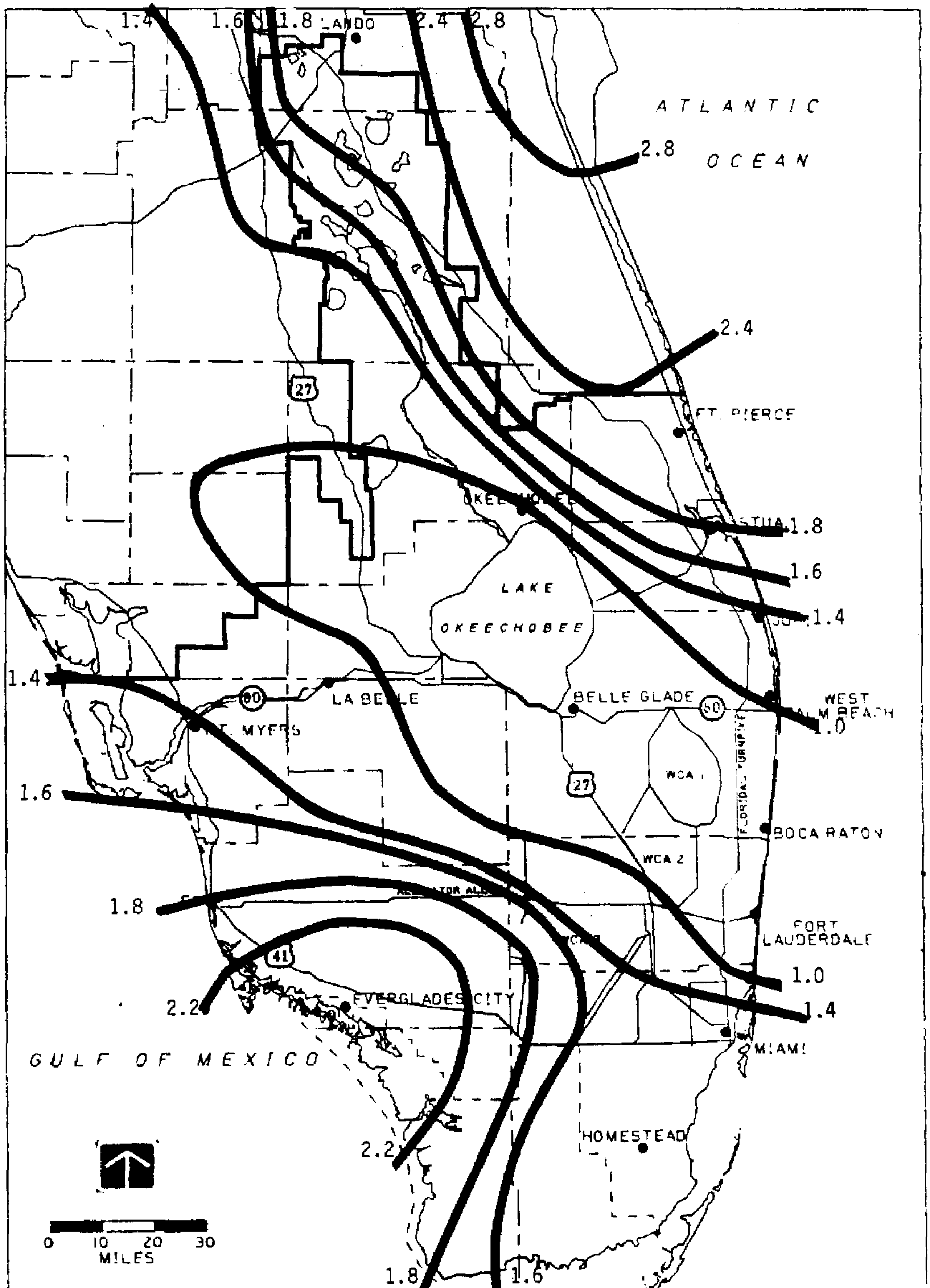
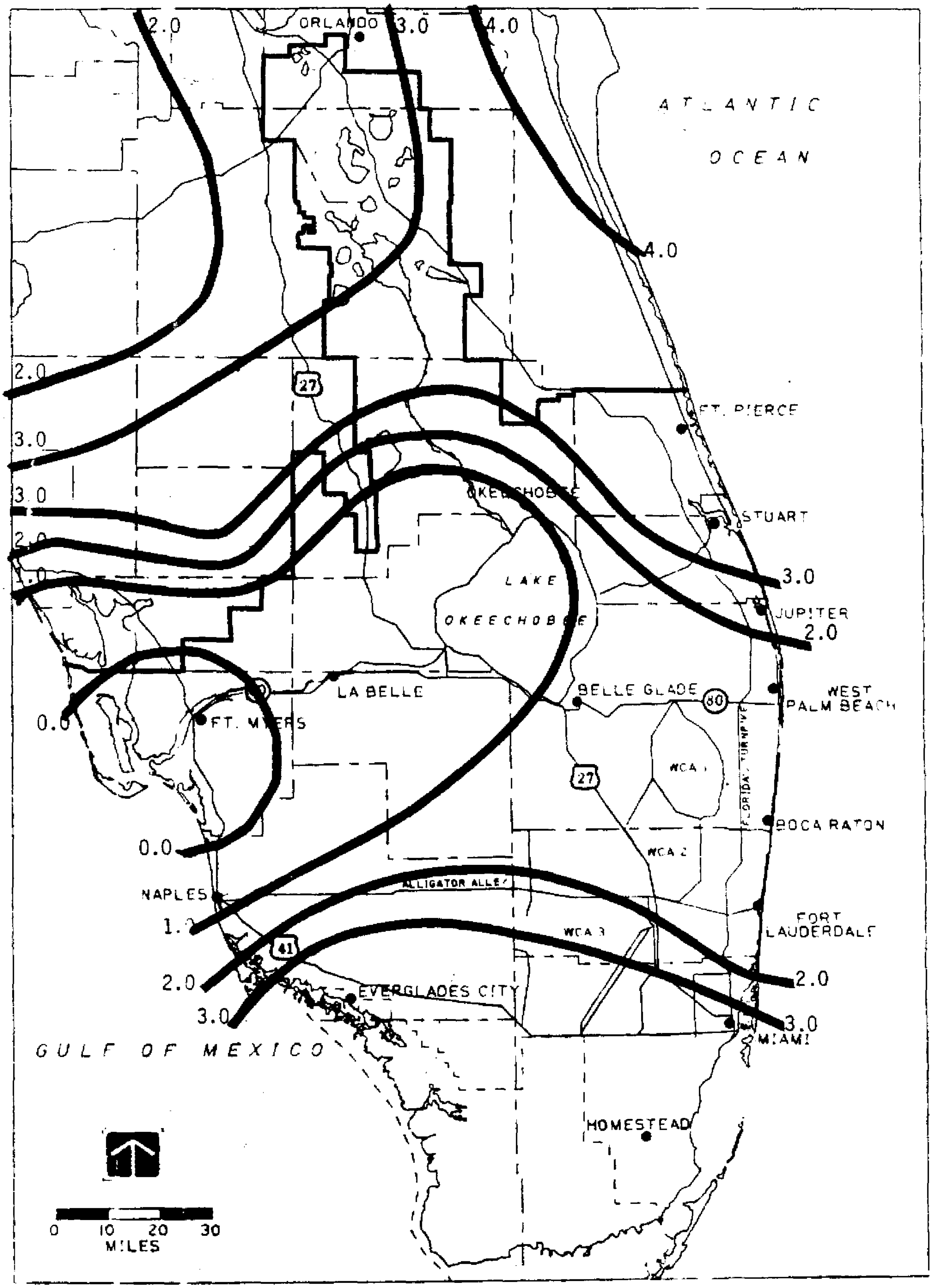


FIGURE 21. t VALUES FOR OVER 2 IN/DAY RAINFALL CHANGE. BREAK AT 1970.
t>1.6 INDICATES A SIGNIFICANT CHANGE OF MEAN AT 90% CONFIDENCE LEVEL.



IGURE 22. YEARLY MEAN DECREASE OF OVER 3 IN/DAY RAINFALL, INCHES. BREAK AT 1970.

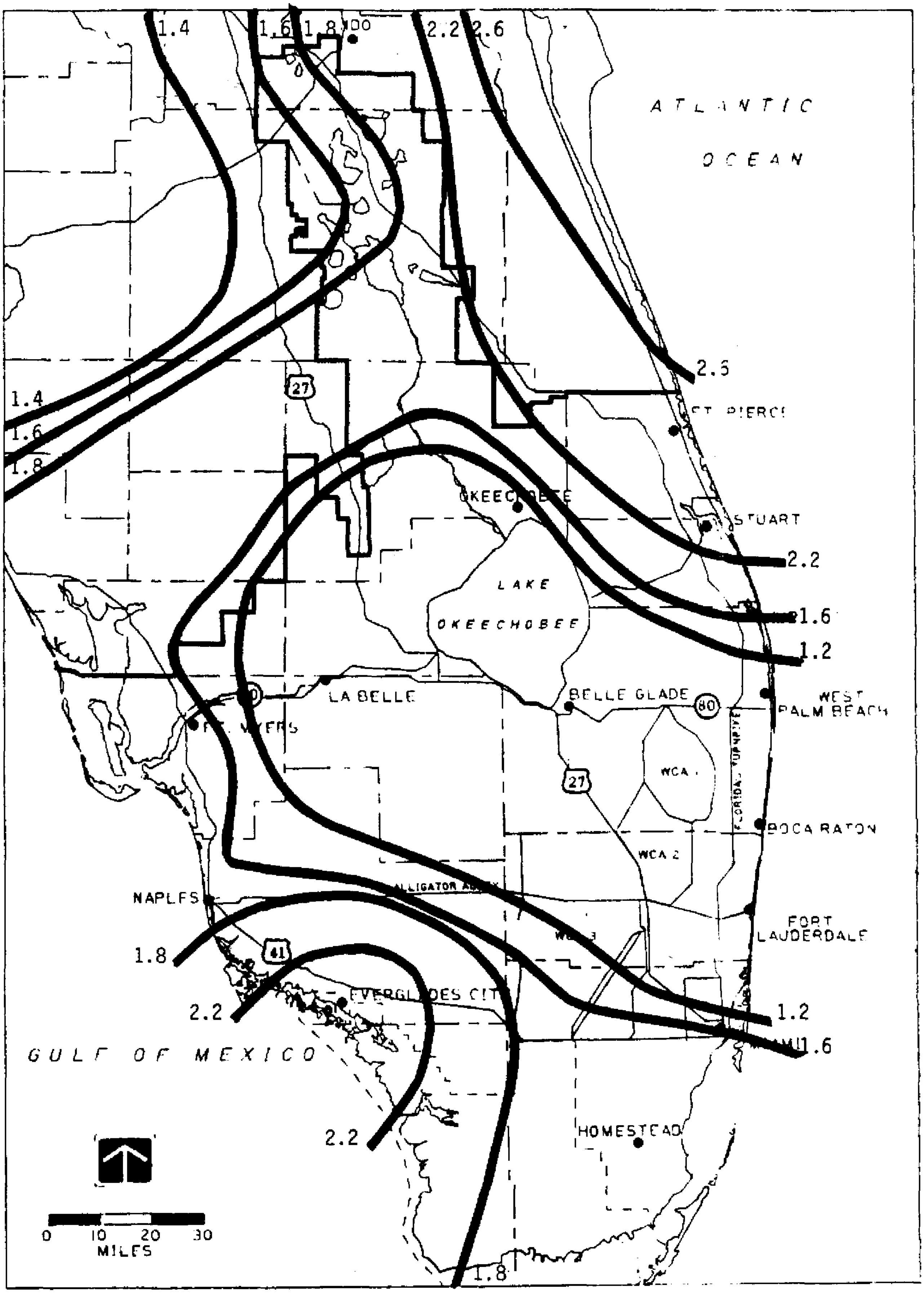


FIGURE 23. t VALUES FOR OVER 3 IN/DAY RAINFALL CHANGE. BREAK AT 1970.
t>1.6 INDICATES A SIGNIFICANT CHANGE OF MEAN AT 90% CONFIDENCE LEVEL.

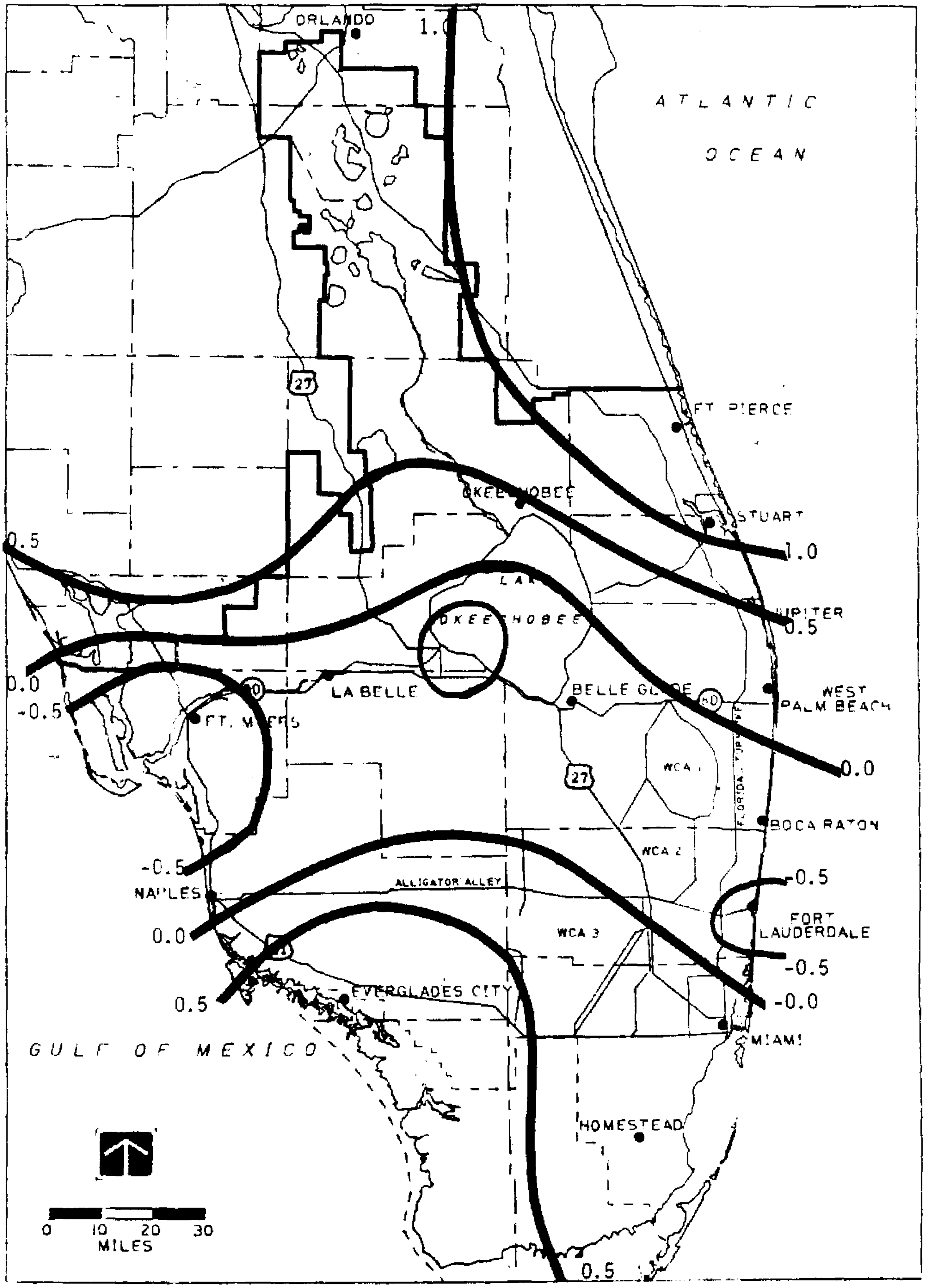


FIGURE 24. DECREASE OF MEAN DAILY MAXIMUM RAINFALL, INCH. BREAK AT 1970.

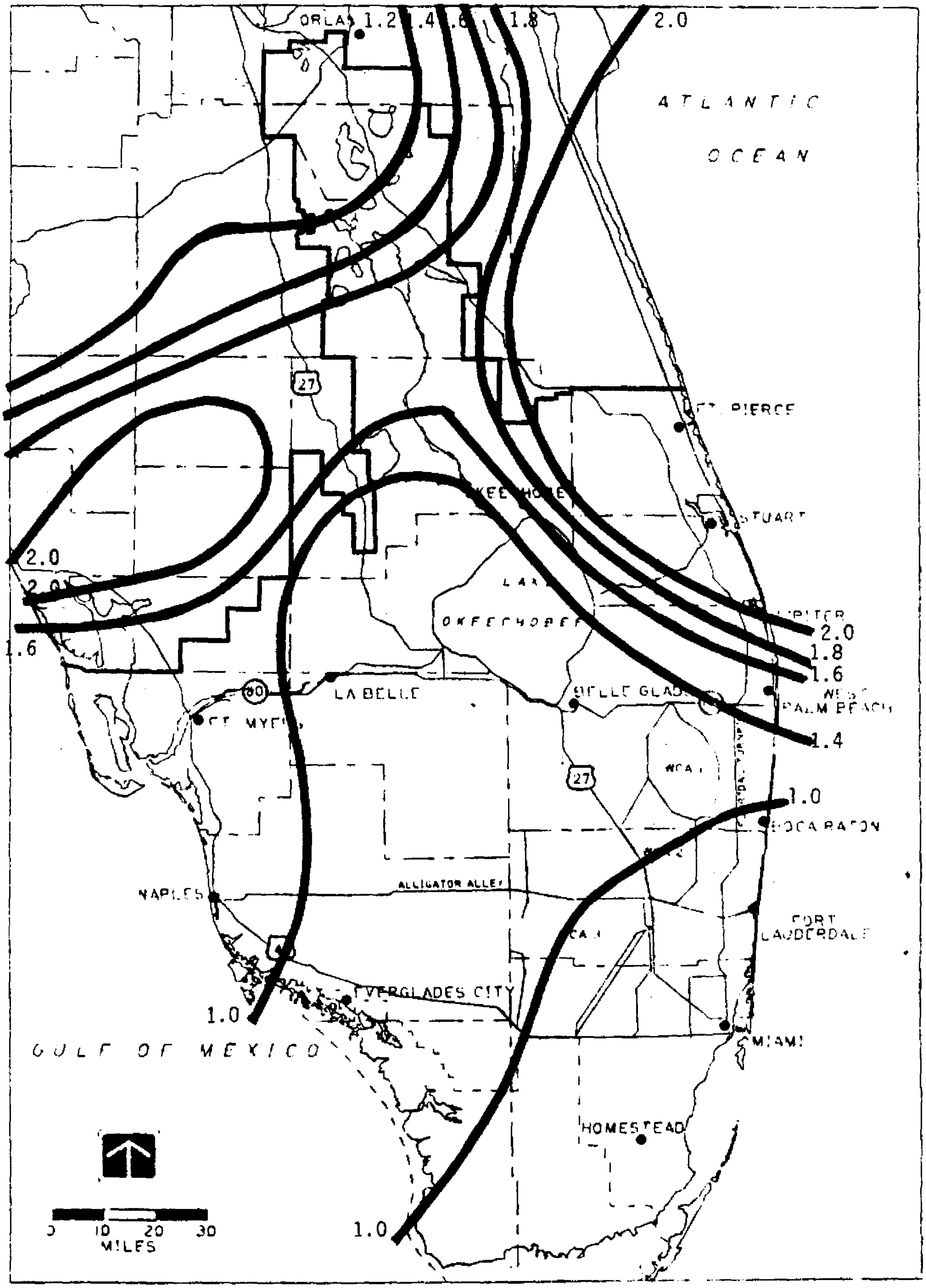


FIGURE 25. t VALUES FOR MEAN DAILY MAXIMUM DIFFERENCE, BREAK AT 1970. T >1.6 INDICATES THERE WAS A SIGNIFICANT CHANGE OF THE MEAN

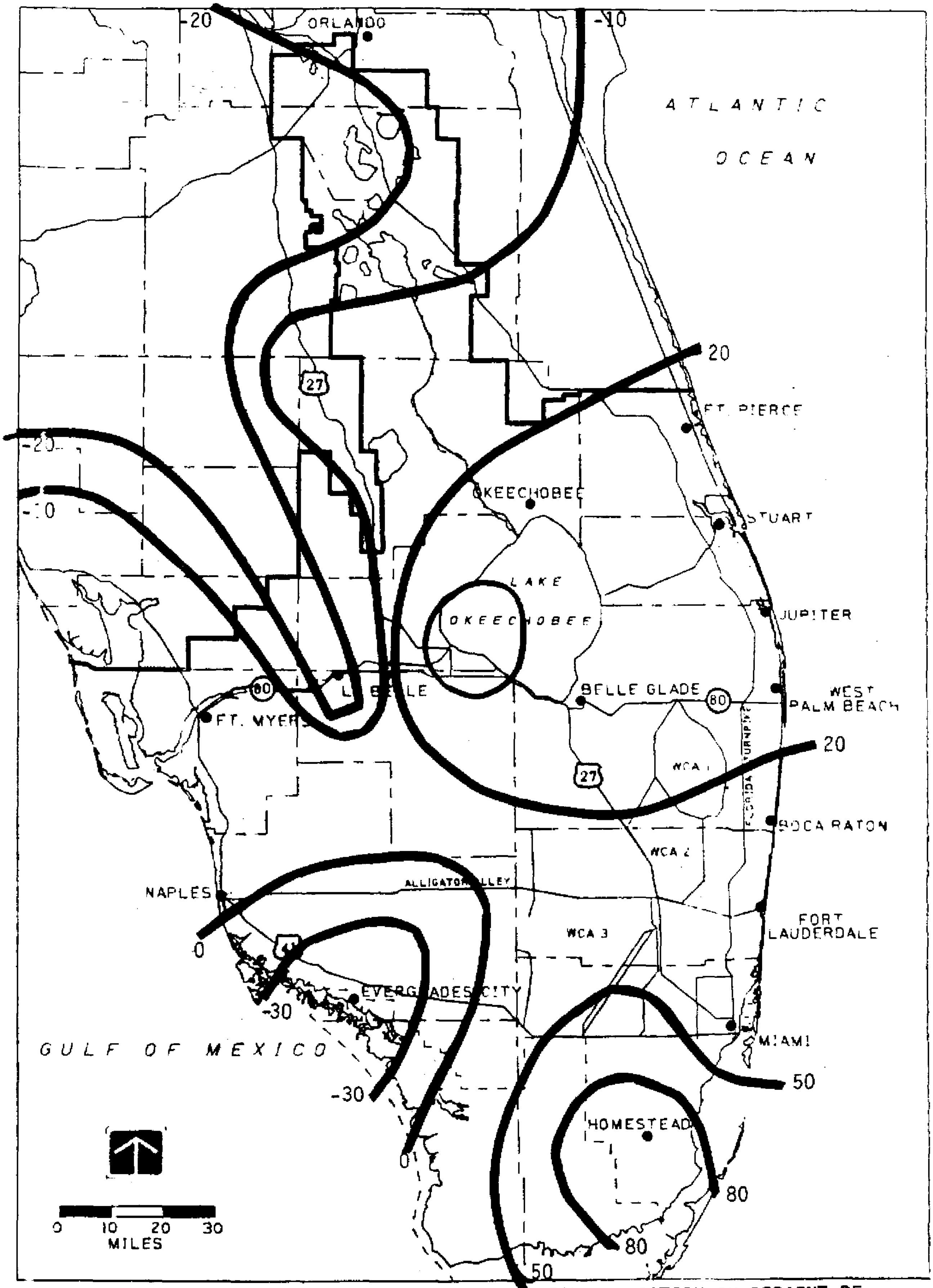


FIGURE 26. INCREASE OF MEAN DAILY MAXIMUM RAINFALL VARIATION IN PERCENT OF VARIATION COEFFICIENT. BREAK AT 1970.

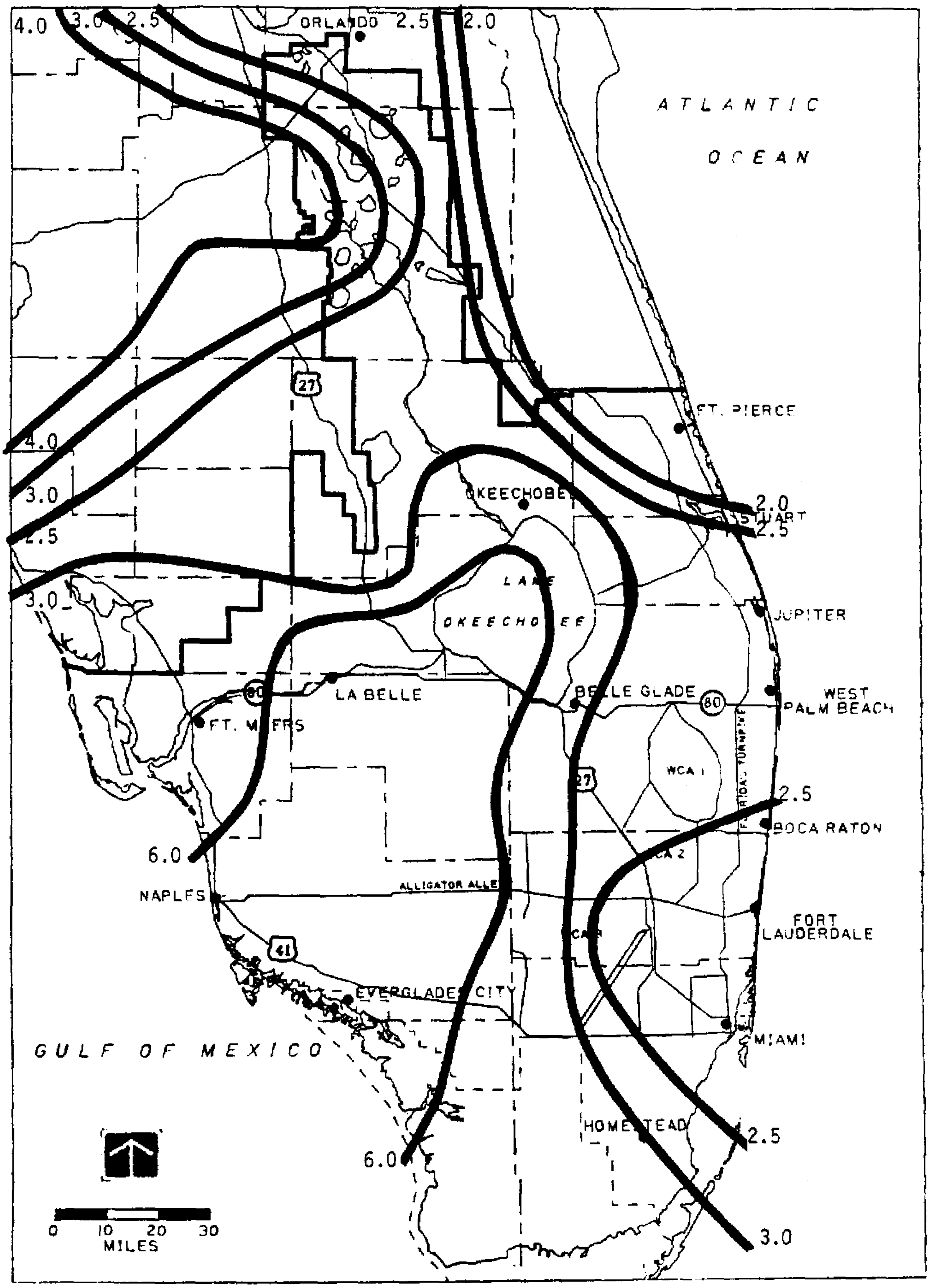


FIGURE 27. F VALUES, MEAN DAILY MAXIMUM RAINFALL VARIATION CHANGE, BREAK AT 1970. F>2.5 INDICATES THERE WAS A SIGNIFICANT CHANGE IN VARIANCE.

LITERATURE CITED

1. Lecture notes for the computer workshop in "Statistical Hydrology", 1978, July 17-21, by Jose D. Salas, Vujica Yevjevich, Duane C. Bees, Jacques W. Delleur, John C. Schaake, Thomas E. Coley, Ertugrul Benzeden, and Richardo A. Smith from the Department of Civil Engineering, Colorado State University.

2. Donald Brandes, 1981, "The Significance of Tropical Cyclone Rainfall in the Water Supply of South Florida". Ph.D. Dissertation, University of Florida.

APPENDIX

COMPUTER PROGRAMS TO DETECT TIME OF CHANGE

```
      PROGRAM CHANGE2(F1,F2,F6,TAPE1=F1,TAPE2=F2,TAPE6=F6)
C     TO DETECT YEAR OF JUMP IN THE MEAN BY CSU PROGRAMS
      DIMENSION X(1000),F(1000),D(1000),T(1000),D2(1000)
     $ ,H(1000),D1(1000)
      CHARACTER*4 XLAB1,XLAB2,YLAB,ID,JD
      DATA XLAB1,XLAB2,YLAB,ID/'TIME','UNIT','PROB','    '/
   10 I=1
      READ(1,20,END=100) ID,IY,X(I)
   20 FORMAT(1X,A4,I3,78X,F5.2)
   30 I=I+1
      READ(1,20,END=100) JD,JY,X(I)
      IF(ID .EQ. JD) GO TO 30
      BACK SPACE 1
      N=I-1
      CALL CHANG2(6,N,X,.9,F,AMEAN,AMODE,TLWR,TUPR,D,AMEND,
     $ TLW,TUP,D2,1,D1,H)
      CHF=0.
      ICH=0
      DO 40 I=1,N
      IF(F(I) .GT. CHF) ICH=I
      IF(F(1) .GT. CHF) CHF=F(I)
   40 CONTINUE
      ICH=IY+ICH-1
      WRITE(2,50) ID,ICH,CHF
   50 FORMAT(A4,I6,F10.4)
      GO TO 10
  100 STOP
      END


C ***********************************************************************
C    THIS ROUTINE WAS DEVELOPED AND PRESENTED IN THE LECTURE NOTES FOR
C    THE COMPUTER WORKSHOP IN STATISTICAL HYDROLOGY HELD JULY 17-21,
C    1978 AT COLORADO STATE UNIVERSITY.  ROUTINE WAS KEY PUNCHED FROM
C    THE LISTINGS IN THIS MANUAL AND MODIFIED TO BE COMPATIBLE WITH
C    FORTRAN ON THE HP3000 COMPUTER DURING 1978 AND 1979.  THIS FORM
C    WAS CHANGED TO CDC COMPATABLE FORTRAN 5 IN 1981.  CONVERSION AND
C    TESTING WAS DONE BY RON MIERAU, SOUTH FLORIDA WATER MANAGEMENT
C    DISTRICT.   SOME ROUTINES WERE SUPERCEEDED BY A LATER MAGNETIC
C    TAPE VERSION FROM A SIMILAR WORKSHOP HELD IN 1980.  SIGNIFICANT
C    IMPROVEMENTS IN ARIMA MODELING WERE MADE IN THE SECOND VERSION
C    AS WELL AS INCLUDING DISAGGREGATION MODELING. THE SECOND VERSION
C    DID NOT INCLUDE THE SET OF ROUTINES DEALING WITH FILLING MISSING
C    DATA,FREQUENCY ANALYSIS AND MANY GOODNESS OF FIT TESTS
C ***********************************************************************
       SUBROUTINE CHANG2 (LU,N,X,ALPHA,F,AMEAN,AMODE,TLWR,TUPR,D,AMEND,
      1 TLW,TUP,D2,IWR,D1,H )
C.......................................................................
C      DETECTING CHANGES.  CASE OF INDEPENDENCE.  POINT OF CHANGE UNKNOWN
```

```
C
C     X     = SAMPLE SERIES OF SIZE N
C     ALPHA = CONFIDENCE LEVEL FOR THE BAYESIAN INTERVAL ESTIMATE
C     F     = POSTERIOR DISTRIBUTION OF POINT OF CHANGE
C     AMEAN = MEAN OF THE DISTRIBUTION F
C     AMODE = MODE OF THE DISTRIBUTION F
C     TUPR = UPPER LIMIT FOR BAYESIAN ESTIMATE OF POINT OF CHANGE
C     TLWR = LOWER LIMIT FOR BAYESIAN ESTIMATE OF POINT OF CHANGE
C     D     = POSTERIOR DISTRIBUTION OF AMOUNT OF CHANGE
C     AMEND = MEAN OF THE DISTRIBUTION D
C     TUP = UPPER LIMIT FOR BAYESIAN ESTIMATE OF AMOUNT OF CHANGE
C     TLW = UPPER LIMIT FOR BAYESIAN ESTIMATE OF AMOUNT OF CHANGE
C     IWR = 0, DO NOT WRITE RESULTS     IWR = 1, WRITE RESULTS
C     DEVELOPED BY DUANE C. BOES, RICARDO A. SMITH, AND JOSE D. SALAS
C     HYDROLOGY AND WATER RESOURCES PROGRAM, COLORADO STATE UNIVERSITY
C     ADAPTED FOR HP3000 BY MIERAU
C.....................................................................
      DIMENSION X(N), F(0:N), D(0:900), H(N), D1(N), D2(0:900)
      DOUBLE PRECISION SUMA,SST
      REAL MULT
      SUM1 = 0.0
      SUM2 = 0.0
      EX = FLOAT(N-2)/2.
      EX1=FLOAT(N-1)/2.
      DO 10  J=1,N
   10 SUM1 = SUM1+X(J)
      AMEA = SUM1/FLOAT(N)
      DO 20  J=1,N
   20 SUM2 = SUM2+(X(J)-AMEA)**2
      N1 = N-1
      DO 70  J=1,N1
      SUM = 0.0
      SUMA = 0.0
      DO 30  I=1,J
   30 SUM = SUM+X(I)
      AMENT = SUM/FLOAT(J)
      J1=J+1
      DO 40  I=J1,N
   40 SUMA = SUMA+X(I)
      AMENNT = SUMA/FLOAT(N-J)
      D1(J) = AMENNT-AMENT
      SUM = 0.0
       SUMA = 0.0
       DO 50  I=1,J
    50 SUMA = SUMA+(X(I)-AMENT)**2
       DO 60  I=J1,N
    60 SUM = SUM+(X(I)-AMENNT)**2
       H(J) = SUM+SUMA
       AJ1 = J
       AJ2 = N - J
       AN = FLOAT(N)/(AJ1*AJ2)
       AN = SQRT(AN)
    70 F(J) = AN*((SUM2/(SUM+SUMA))**EX)
       SUM = 0.0
       AMEAN = 0.0
```

```
      DO 74  J=1,N1
   74 SUM = SUM+F(J)
      DO 78  J=1,N1
   78 F(J) = F(J)/SUM
      AMEAN = 0.0
      DO 80  J=1,N1
   80 AMEAN = AMEAN+J*F(J)
      AMODE = 1.0
      AMO = F(1)
      DO 100  J=1,N1
      IF (AMO-F(J)) 90,100,100
   90 AMO = F(J)
      AMODE = J
  100 CONTINUE
      ALPHA1=(1.-ALPHA)/2.
      K=0
      F(K) = 0.0
      SUM = 0.
      SUM1 = 0.
      DO 110  J=1,N1
      SUM = SUM+F(J)
      SUM1=SUM1+F(J-1)
      IF (SUM.LT.ALPHA1) GO TO 110
      TLWR=FLOAT(J)-(SUM-ALPHA1)/(SUM-SUM1)
      GO TO 120
  110 CONTINUE
  120 SUMA = 0.0
      SUMA1 = 0.0
      F(N)=0.
      DO 130 J=1,N1
      JJ = N1-J+1
      SUMA = SUMA+F(JJ)
      SUMA1=SUMA1+F(JJ+1)
      IF (SUMA.LT.ALPHA1) GO TO 130
      TUPR=FLOAT(JJ)+(SUMA-ALPHA1)/(SUMA-SUMA1)
      GO TO 140
  130 CONTINUE
  140 XMAX = X(1)
      DO 150  J=2,N
  150 XMAX = AMAX1(XMAX,X(J))
      XMIN = X(1)
      DO 160  J=2,N
  160 XMIN = AMIN1(XMIN,X(J))
            MULT = 1.
            MULTN=1
            MULTX=1
            DO 180 KK = 1,2
      SUM=-(XMAX-XMIN)
      DELTAX=0.0
      DO 180 K=1,100
      SUMA = 0.0
```

```
      SUM=SUM+DELTAX
      D2(K)=SUM
      DO 170  J=1,N1
      EJ = J
      EN = N
      ENJ = EJ*(EN-EJ)
      SS=H(J)+ENJ*((D2(K)-D1(J))**2)/FLOAT(N)
            SST = (1./SS) * MULT
C           IN THE ORIGINAL VERSION AN UNDERFLOW PROBLEM OCCURED
C           WHEN BOTH SS AND EX1 WERE LARGE.  D(K) LOST ALL SIGNIFICANT
C           FIGURES IN THIS SITUATION AND THE RATIO OF D(K) TO THE
C           SUM OF ALL D(K) BECAME INDETERMINATE.  THIS SITUATION WAS
C           CORRECTED BY FINDING A COMMON MULTIPLIER TO KEEP
C           SST**EX1 IN COMPUTABLE RANGE. STATEMENTS ADDED TO ACCOMPLISH
C           THIS ARE INDENTED
C
            IF(KK.EQ.2) GO TO 170
            MULTT=1
            SSTT=SST
            DO 165 I = 1,100
            IF(SSTT .LT. 0.1) MULTT = MULTT + 1
            IF(SSTT .LT. 0.1) SSTT= SSTT*10.
            IF (SSTT .GE. 0.1) GO TO 166
             SST=SSTT
  165       CONTINUE
  166       IF (MULTT .GT. MULTX) MULTX = MULTT
            IF (MULTT .LT. MULTN) MULTN = MULTT
            IF (MULTN .EQ. 1) MULTN = MULTX
            IF (K .EQ. 100) MULT=10.**((MULTX+MULTN)/2 - 1)
      IF (K .EQ. 100  .AND. MULTX .EQ. 1) MULT=1.
  170 SUMA=SUMA+SST**EX1
      D(K) = SUMA
  180 DELTAX = 2*(XMAX-XMIN)/100.
      SUM = 0.0
      DO 190 J=1,100
  190 SUM = SUM+D(J)
      DO 195 J=1,100
  195 D(J) = D(J)/SUM
      AMEND = 0.0
      DO 200 K=1,100
  200 AMEND = AMEND+D2(K)*D(K)
      K=0
      D(K) = 0.0
      D2(K) = 0.0
      SUM = 0.0
      SUM1 = 0.0
      DO 210 J=1,100
      SUM = SUM+D(J)
      SUM1=SUM1+D(J-1)
      IF (SUM.LT.ALPHA1) GO TO 210
      JJ = J-1
      TLW=D2(J)-(D2(J)-D2(JJ))*(SUM-ALPHA1)/(SUM-SUM1)
      GO TO 220
```

```
210 CONTINUE
220 D(101)=0.0
    SUMA1 = 0.0
    SUMA = 0.
    D2(101) = 0.
    DO 230 J=1,100
    JJ=100-J+1
    SUMA1=SUMA1+D(JJ+1)
    SUMA = SUMA+D(JJ)
    IF (SUMA.LT.ALPHA1) GO TO 230
    J2 = JJ+1
    TUP=D2(JJ)+(D2(J2)-D2(JJ))*(SUMA-ALPHA1)/(SUMA-SUMA1)
    GO TO 240
230 CONTINUE
240 IF  (IWR.EQ.0) RETURN
    WRITE (* ,250)
250 FORMAT (1H1///5X,"DETECTING CHANGES IN A GIVEN SERIES USING BAYESI
   1AN ANALYSIS",//5X,"POSTERIOR DISTRIBUTION OF TIME OF CHANGE",//5X,
   2"CHANGE",10X,"DISTRIBUTION",/)
    DO 260  J=1,N1
260 WRITE (* ,270)  J, F(J)
270 FORMAT (6X,I3,12X,F8.3)
    WRITE (* ,280) AMEAN, AMODE
280 FORMAT(/5X,"MEAN OF THE DISTRIBUTION =",F8.3/5X,"MODE OF THE DISTR
   1IBUTION =",F8.3/)
    WRITE (* ,290)  TUPR, TLWR, ALPHA
290 FORMAT (/5X,"UPPER BAYESIAN LIMIT =",F8.3/5X,"LOWER BAYESIAN LIMIT
   1 =",F8.3/5X,"CONFIDENCE LEVEL      =",F8.3)
    WRITE (LU,300)
300 FORMAT(//5X,"POSTERIOR DISTRIBUTION OF AMOUNT OF CHANGE",//5X,"AMO
   1UNT ",10X,"DISTRIBUTION",/)
    DO 310 J=1,100
310 WRITE (LU,320) D2(J), D(J)
320 FORMAT (4X,F8.3,10X,F8.3)
    WRITE (LU,330) AMEND
330 FORMAT (/5X,"MEAN OF THE DISTRIBUTION =",F8.3/)
    WRITE (LU,340) TUP, TLW, ALPHA
340 FORMAT (/5X,"UPPER BAYESIAN LIMIT =",F8.3/5X,"LOWER BAYESIAN LIMIT
   1 =",F8.3/5X,"CONFIDENCE LEVEL      =",F8.3/)
    RETURN
    END
```